Research Report 362

Riikka Mononen

Early mathematics interventions

Supporting young children with low performance in mathematics

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Abstract
The purpose of this thesis was to investigate the effectiveness of early mathematics interventions for young children with low performance in mathematics. Previous research has indicated that early mathematics skills are a strong predictor of later mathematics performance. The goal of early mathematics support by means of interventions is to improve mathematics performance, and consequently, to diminish the possibility of mathematics learning disability emerging later on. This thesis sought to complement and extend previous research in the field of early mathematics interventions, by reviewing early mathematics interventions, and investigating the effectiveness of two early mathematics intervention programmes.

Study I reviewed mathematics interventions (*N* = 19) aimed at 4–7-year-old children with low performance in mathematics. For each intervention, effect sizes were calculated for mathematics outcome measures, and the pedagogical implementation was described. The effectiveness of the RightStart Mathematics (RS) (Cotter, 2001) instruction was investigated in Studies II and III. In Study II, the instruction was provided for Finnish kindergartners (RS group: *n* = 38, comparison group: *n* = 32) in general education classrooms, with focus on low-performing children. In Study III, the RS instruction was provided in special education classrooms for children with a specific language impairment (SLI group: *n* = 9, comparison group: *n* = 32). In Study IV, a mathematics intervention programme Improving Mathematics Skills in the Second Grade (IMS-2) (Mononen & Aunio, 2012) was developed, and its effectiveness for second graders performing low in mathematics was examined (IMS-2 group: *n* = 11, low-performing controls: *n* = 13 and typically performing controls: *n* = 64). In Studies II-IV, quantitative methods were used for analysing the interventions' effects.

According to the results of the review, in the majority of the interventions, the mathematics skills of the participating children improved more than the skills of the children in control groups, with effect sizes varying from small to large. Progress in mathematics learning was evident when

instruction included one or more of the following instructional features: explicit instruction, peer-assisted instruction, applying a concrete-representational-abstract sequence, computer assisted instruction, or games. Study II showed that the RS instruction was as effective as the typical Finnish kindergarten mathematics instruction. The counting skills of the initially low-performing children improved to the level of their typically performing peers. Follow-up in the first grade revealed performance differences between the initially low- and typically performing children, highlighting the importance of continuously monitoring progress, and providing intensified support. In Study III, children with a SLI receiving RS instruction improved their counting skills to the level of their peers. In the first grade follow-up, the children with SLI performed similarly to their peers in addition and subtraction skills (accuracy) and multi-digit number comparison. In Study IV, the mathematics skills of the second graders participating in the IMS-2 intervention did not improve more than the skills of the children in control groups. However, the study provided valuable information about the functionality of the IMS-2 programme's intensity and content.

To conclude, in general, the results indicate that rather than waiting for children to fail, mathematics interventions can be used successfully to promote the early mathematics skills of children with low performance in mathematics, already before the onset of formal schooling and in the early grades. Therefore, identifying low performance in mathematics and providing sufficient support should be emphasised already in early childhood education, in accordance with the Finnish three-tiered educational support system.

Keywords: early mathematics skills, low performance in mathematics, mathematics learning disability, mathematics intervention, review, specific language impairment, educational support

Helsingin yliopisto, Käyttäytymistieteellinen tiedekunta
Opettajankoulutuslaitos
Tutkimuksia 362

Riikka Mononen

Matemaattiset interventiot
Matemaattisilta taidoiltaan heikkojen lasten tukeminen ennen koulunaloitusta ja ensimmäisinä kouluvuosina

Tiivistelmä
Tämän väitöstutkimuksen tarkoituksena oli tarkastella matemaattisten interventioiden vaikuttavuutta lapsilla, joilla on heikot matemaattiset taidot. Aiempi tutkimus on osoittanut, että matemaattisilla taidoilla ennen kouluikää on vahva yhteys myöhempään matematiikan osaamiseen. Varhaisella matemaattisten taitojen tukemisella pyritään kohentamaan matemaattisen osaamisen tasoa ja siten ennaltaehkäisemään matematiikan oppimisvaikeuksien syntymistä. Tämä tutkimus täydentää ja laajentaa nykyistä tietämystä matemaattisista interventioista kirjallisuuskatsauksella matemaattisista interventioista, jotka on suunnattu matemaattisesti heikkojen lasten tukemiseen. Lisäksi tutkittiin kahden matemaattisen interventio-ohjelman vaikuttavuutta.

Osatutkimuksen I kirjallisuuskatsaus tarkasteli matemaattisia interventioita (N = 19), jotka oli suunnattu 4–7-vuotiaille lapsille, joilla on heikot matemaattiset taidot. Jokaisen intervention matemaattisten taitojen osaamistuloksille laskettiin efektikoot ja intervention pedagoginen toteutus kuvailtiin. Osatutkimuksissa II ja III tutkittiin RightStart Matematiikka (RS) (Cotter, 2001) ohjelman vaikuttavuutta. Osatutkimuksessa II RS-opetusta annettiin suomalaisille esiopetusikäisille lapsille (RS-ryhmä: n = 38, verrokkiryhmä: n = 32) yleisopetuksen esiopetusryhmissä. Tarkastelun kohteena olivat myös matemaattisilta taidoiltaan heikot lapset. Osatutkimuksessa III RS-opetusta annettiin erityisopetuksen ryhmissä lapsille, joilla oli kielellinen erityisvaikeus (SLI-ryhmä: n = 9, verrokkiryhmä: n = 32). Osatutkimuksessa IV kehitettiin interventio-ohjelma Matemaattisten taitojen tukeminen toisella luokalla (MTT-2) (Mononen & Aunio, 2012) ja sen vaikuttavuutta tutkittiin toisluokkalaisilla, joilla oli heikot matemaattiset taidot (MTT-2-ryhmä: n = 11, heikot verrokkiryhmä: n = 13, tavanomaisesti osaavat verrokkiryhmä: n = 64). Osatutkimuksissa II-IV interventioiden vaikuttavuutta analysoitiin määrällisillä tutkimusmenetelmillä.

Kirjallisuuskatsauksen perusteella suurin osa matemaattisista interventioista näytti kohentavan taidoiltaan heikkojen lasten matemaattisia

taitoja, efektikokojen vaihdellessa pienestä suureen. Taidot paranivat selkeästi, kun interventiossa käytettiin yhtä tai useampaa seuraavista opetustavoista: eksplisiittinen opetus, oppilasparityöskentely, opetuksen eteneminen konkreettinen-kuva-abstrakti -jatkumolla, tietokoneavusteinen opetus tai pelit. Osatutkimus II osoitti, että RS-opetus oli yhtä tehokasta kuin tavanomainen suomalainen esiopetuksen matematiikan opetus. Esiopetusvuoden jälkeen heikkojen lasten laskemisen taidot olivat nousseet samalle tasolle tavanomaisesti osaavien kanssa. Ensimmäisellä luokalla taitoeroja esiintyi jälleen alun perin heikkojen ja tavanomaisesti osaavien lasten välillä. Tämä tulos korostaa sitä, että lasten matemaattisia taitoja tulisi arvioida säännöllisesti, ja lisätukea tulisi tarjota niille, jotka sitä näyttävät tarvitsevan. Osatutkimuksessa III RS-opetusta saaneiden lasten, joilla oli kielellinen erityisvaikeus, laskemisen taidot olivat kohentuneet tavanomaisesti osaavien lasten tasolle esiopetusvuoden lopussa. Ensimmäisellä luokalla nämä lapset suoriutuivat samalla tasolla tavanomaisesti osaavien lasten kanssa yhteen- ja vähennyslaskuissa (oikeellisuus) sekä moninumeroisten lukujen vertailussa. Osatutkimuksessa IV MTT-2 interventiota saaneiden matemaattisilta taidoiltaan heikkojen toisluokkalaisten osaamisen kasvu matemaattisissa taidoissa ei eronnut verrokkiryhmien lasten taitojen osaamisen kasvusta. Tämä osatutkimus antoi arvokasta tietoa MTT-2 intervention toimivuuteen vaikuttavista asioista, kuten interventio-ohjelman sisällöstä ja kestosta.

Yhteenvetona voidaan todeta, että yleisesti ottaen matemaattisten interventioiden avulla voidaan tukea onnistuneesti jo ennen koulunaloitusta ja ensimmäisinä kouluvuosina niitä lapsia, jotka ovat matemaattisilta taidoiltaan heikkoja. Tästä syystä, ja suomalaista kolmiportaisen tuen mallia mukaillen, lasten heikot matemaattiset taidot tulisi pystyä tunnistamaan arvioinnin keinoin jo ennen koulunaloitusta ja tarjota tukea sitä tarvitseville.

Avainsanat: esi- ja alkuopetus, heikko matemaattinen osaaminen, kielellinen erityisvaikeus, kirjallisuuskatsaus, matemaattiset interventiot, matemaattiset taidot, matematiikan oppimisvaikeudet

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Helsinki, November, 2014
Riikka Mononen

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List of tables

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Original articles

This thesis is based on the following four original publications, which are referred to in the text by their Roman numerals (Studies I – IV):

Study I: Mononen, R., Aunio, P., Koponen, T., & Aro, M. (2014). A review of early numeracy interventions for children at risk in mathematics. *International Journal of Early Childhood Special Education, 6*(1), 25–54. http://www.int-jecse.net/files/Z70G8EJKQ63RAZ9O.pdf.

Study II: Mononen, R., Aunio, P., & Koponen, T. (2014a). Investigating RightStart Mathematics kindergarten instruction in Finland. *Journal of Early Childhood Education Research, 3*(1), 2–26. http://jecer.org/wp-content/uploads/2014/04/Mononen-Aunio-Koponen-issue3-1.pdf.

Study III: Mononen, R., Aunio, P., & Koponen, T. (2014b). A pilot study of the effects of RightStart instruction on early numeracy skills of children with specific language impairment. *Research in Developmental Disabilities, 35*(5), 999–1014.doi: 10.1016/j.ridd.2014.02.004.

Study IV: Mononen, R., & Aunio, P. (2014). A Mathematics intervention for low-performing, Finnish second graders: findings from a pilot study. *European Journal of Special Needs Education*. Advance online publication. doi: 10.1080/08856257.2014.922794.

1 Introduction

Early childhood education has a major role in many countries in providing children with quality early experiences in learning mathematics. The aim is that in the beginning of formal schooling and in the early grades most children would have gained adequate foundational mathematics skills to be able to learn more advanced school mathematics. There is already a wide variation in children's early mathematics performance before formal schooling (e.g., Aubrey, Dahl, & Godfrey, 2006; Aunio & Niemivirta, 2010; Navarro et al., 2012; Stock, Desoete, & Roeyers, 2009a). Recent studies on Finnish kindergartners and first graders (Mononen & Aunio, 2013; Mononen, Aunio, Hotulainen, & Ketonen, 2013)[1] showed that at the beginning of the school year, there is a small group of children with significantly weaker mathematics skills compared to their peers, and there is a group of children who have already mastered most of the content to be taught during the upcoming school year. The same trend has been observed in international studies (Engel, Claessens, & Finch, 2013; Wright, 1991). Longitudinal studies (e.g., Aunola, Leskinen, Lerkkanen, & Nurmi, 2004) have indicated that the mathematics performance gap between the low- and typically performing children tends to increase over the years, if adequate support is not provided for those children who show low performance. In order to meet the diversity in children's learning support needs, legal and practical actions to develop educational support systems have been made (Finland's Basic Education Act 628/1998, Amendment 642/2010; Gersten, Beckmann, et al., 2009). In several countries, including Finland, the method for delivering educational support for all children is organized on a three-tiered model: (1) general support, (2) intensified support and (3) special support (Finland's Basic Education Act 628/1998, Amendment 642/2010; Lembke, Hampton, & Beyers, 2012). Along with implementing the three-tiered model of support, there have been growing demands to provide educators with evidence-based instruction materials and methods, in order to ensure the best available instruction for improving children's skills.

The focus of this thesis is on young children with low performance in mathematics and how their mathematics development can be supported

[1] In this thesis children having instruction a year before the beginning of their formal schooling, or first grade, are referred to as kindergartners. Depending on the age the children start their first grade (country specific differences), kindergartners are from 5 to 6 years old.

by means of early mathematics interventions.[2] There is evidence that mathematics interventions for school-aged children performing low in mathematics or with learning disabilities have been beneficial in increasing the level of mathematics performance (e.g., Gersten, Chard, et al., 2009; Kroesbergen & Van Luit, 2003; Kunsch, Jitendra, & Sood, 2007). However, there is proof that by using appropriate and validated mathematics measures, children's weak performance in mathematics can already be identified before the beginning of formal schooling (Aunio, Hautamäki, Heiskari, & Van Luit, 2006; Stock et al., 2009a; Weiland et al., 2012). Therefore, in accordance with the three-tiered model of support, rather than waiting for these children's mathematics skills to lag severely behind those of their peers, children performing low in terms of their early mathematics skills should be provided with the earliest possible support. Ultimately, the aim of early support is to reduce the emergence of mathematics learning disability. Currently, there is little knowledge about the overall effectiveness of early mathematics interventions for children performing low in mathematics. Likewise, only few mathematics intervention studies have been conducted in Finland.

This thesis seeks to complement and extend previous research in the field of mathematics interventions by reviewing early mathematics interventions for children with low performance in mathematics and investigating the effectiveness of two different early mathematics interventions. More specifically, in the review, the interventions were examined in terms of their effectiveness and pedagogical implementation (Study I). The RightStart Mathematics (Cotter, 2001) core curricular instruction was introduced to Finnish kindergartners both in general education classes (Tier I) and in special education classes (Tier III) for children with a specific language impairment (Studies II and III, respectively). For second graders performing low in mathematics, a mathematics intervention called Improving Mathematics Skills in the Second Grade (Mononen & Aunio, 2012) was developed as part of the ThinkMath project and the intervention's effectiveness was examined in intensified support (Tier II) with small-groups (Study IV).[3] A longitudinal pre-post control design was applied in all the intervention studies in order to see the impact of intervention on the children's mathematics development immediately and several months after the intervention.

[2] In this thesis intervention is defined according to Tilly and Flugum (as cited in Riley-Tillman & Burns, 2009, p. 2) as "a planned modification of the environment made for the purpose of altering behavior in a prespecified way".

[3] The ThinkMath project seeks to develop evidence-based mathematics and thinking skills interventions for children from kindergarten to second grade. It is conducted under the University of Helsinki, Department of Teacher Education, Special Education, and funded by the Finnish Ministry of Education and Culture, for 2011-2015.

The present thesis consists of two parts. The first part includes a theoretical introduction, the aims and methods used, an overview of the original studies, the main findings and a discussion. The aim of the introduction is to provide a theoretical frame for addressing why, how and what kind of mathematics interventions should be provided for children with low performance in mathematics. Accordingly, early mathematics core skills and individual differences in mathematics development as well as factors that may influence the mathematics development (i.e., domain-general and domain-specific skills, and learning environment) will be presented. This is followed by how children's mathematics skills can be supported in educational settings by means of interventions. Furthermore, beneficial instructional features to support children with low mathematics performance are described based on the findings from mathematics intervention review studies in school-aged children. The introduction chapter ends with a description of the aims and overview of the methods used in the original studies. In the following chapter an overview of four original studies is provided. The final chapter gives a general summary of the results and ends with a discussion of the theoretical and practical implications, limitations of the thesis and future avenues for research. The second part of the thesis consists of the four original articles, which were published in international peer-reviewed journals.

1.1 Early mathematics development

During their early childhood years (considered here as up to eight years of age), most children learn mathematics skills that will provide a foundation for their later more advanced mathematics learning. For describing the mathematics skills that children acquire during their early childhood, many different terms have been used in the literature, such as early mathematics skills (e.g., Sarama & Clements, 2009), early numeracy skills (e.g., Toll & Van Luit, 2014) and number sense (e.g., Jordan, Kaplan, Oláh, & Locuniak, 2006). In this thesis the term early mathematics is applied, as the early mathematics core skills presented in the following mainly refer to the research work by Sarama & Clements (2009).[4]

1.1.1 Early mathematics core skills

There are a number of research-based frameworks developed for describing the development early mathematics skills (e.g., Krajewski as cited in

[4] In Studies I and III, the term early numeracy skills was used; however, the meaning is considered the same as that of the term early mathematics skills.

Krajewski & Schneider, 2009; Sarama & Clements, 2009; Wright, Martland, Stafford, & Stanger, 2002). The one chosen for this thesis is the framework of early mathematics learning trajectories by Sarama and Clements (2009), because it is grounded on a range of research findings by several authors. In the following, early mathematics core skills relevant for the age group of this thesis are operationalised under subheadings according to Sarama and Clements's framework (2009), namely, (1) quantity, number and subitising, (2) verbal and object counting, (3) comparing, ordering and estimating and (4) arithmetic: addition and subtraction (Table 1). As this thesis concentrates on early mathematics core skills, applied skills, such as geometry, spatial thinking and measurement are not addressed here. As research findings from other sources are also included, some of the content of the paragraphs may slightly overlap each other (e.g., Quantity, number and subitising and Comparing, ordering and estimating).

Quantity, number and subitising

Current research suggests that the foundation for mathematics development is grounded in a nonverbal number sense, identified as two systems for tracking quantity: *an exact number system* for small quantities (i.e., subitising) and *an approximate number system* (ANS) for larger quantities (Geary, 2013a; Hornung, Schilz, Brunner, & Martin, 2014). In their framework, Sarama and Clements (2009) concentrate only on subitising skill, but for clarification, the concept of ANS is also introduced here. The exact number system allows for fast and exact recognition of small quantities (1–4), referred to perceptual subitising (Sarama and Clements, 2009). Conceptual subitising, a more developed form of subitising (Sarama & Clements, 2009), is considered when seeing parts of something and putting together the whole entity quickly (e.g., seeing a domino pattern of six dots as three and three). In general, subitising introduces ideas of quantity: cardinality (i.e., the last said number word represents the total number of objects), basic ideas of comparison (i.e., more and less), ideas of parts and wholes and their relationships, and early arithmetic (Clements & Sarama, 2009). ANS, on the other hand, is a mental system that helps us to estimate large quantities approximately (e.g., "Are there are more blue [20] or green dots [45]?" or "There are approximately one hundred dots", if 107 are represented) (Mazzocco, Feigenson, & Halberda, 2011a).

Table 1. Developmental progress in early mathematics core skills (modified on Sarama & Clements, 2009).

<table>
<tr><th></th><th colspan="4">Approximate age (in years)</th></tr>
<tr><th>Skill</th><th>4</th><th>5</th><th>6</th><th>7</th></tr>
<tr><td>Quantity, number and subitising</td><td>- perceptual subitiser to 4
●
●
●
- connections between quantity, number word and number symbol (***, "three", 3) start to develop</td><td>- perceptual subitiser to 5
- conceptual subitiser to 5 – 10
●●●●●
●●</td><td>- conceptual subitiser to 20
●●●●● ●●●
●●●●●</td><td>- conceptual subitiser with place value and skip counting</td></tr>
<tr><td>Verbal and object counting</td><td>- counter to 10 (begins to understand cardinality)
- verbal counter to 20
- producer a group of objects up to 5</td><td>- counter and producer 10+
- counter backwards from 10</td><td>- counter from any number
- skip counter by 10's to 100
- verbal counter to 100
- skip counter by 5's and 2's
- counter of imagined (hidden) items
- counter of quantitative units/ place value
- counter to 200</td><td>- number conserver
- counter forward and backward</td></tr>
<tr><td>Comparing, ordering and estimating</td><td>- nonverbal comparer of dissimilar items
- matching comparer (1–6)
- counting comparer (same size, 1–5)
- mental number line to 5</td><td>- counting comparer (5, dissimilar size)
- ordinal counter (1st–10th)
- spatial extent estimator ('small' and 'big' numbers)
- counting comparer (10)</td><td>- mental number line to 10
- serial orderer to 6+
- spatial extent estimator ('small', 'middle-sized' and 'large' numbers)</td><td>- place value comparer
- mental number line to 100
- estimator of quantities ('about forty')</td></tr>
<tr><td rowspan="3">Arithmetic: Addition and subtraction</td><td colspan="2">- find result for joining and part-part-whole problems (direct modelling with objects, counting-all)
- make it N (make one number to another by counting-on)
- finds change (5 + _= 7) by adding on objects
- composer to 4 and up to 10 (knowing number combinations)</td><td>- part-whole +/- initial understanding</td><td>- problem solver +/- with flexible strategies and known combinations (retrieval of facts)
- composer with tens and ones
- multidigit +/-</td></tr>
<tr><td rowspan="2"></td><td colspan="2">- counting on and counting up-to strategies (+/-)</td><td></td></tr>
<tr><td></td><td colspan="2">- deriver +/- (7 + 7 = 14, so 7 + 8 = 15) (flexible strategies and derived combinations)</td></tr>
</table>

ANS improves gradually from infancy to adulthood, as it has been shown that children succeed in discriminating between increasingly more difficult ratios of numerosities (Mazzocco et al., 2011a). For example, children at around the age of six months can discriminate between sets that differ by a ratio of 2:1 (e.g., fourteen dots vs. seven dots), at around nine months by a ratio of 3:2 (e.g., twelve dots vs. eight dots), at around six years by a ratio of 6:5 (e.g., twelve dots vs. ten dots) and finally, some adults can discriminate by a ratio of 11:10 (e.g., 11 dots vs. 10 dots) (Siegler & Lortie-Forgues, 2014). Geary (2013a) proposed that number words and Arabic numerals have meaning only if they are associated with the quantity they present (e.g., “three”, 3, ***), and ANS may be the foundation for making these associations. Having numerous experiences of naming small collections will help children to build connections between quantity, number words and Arabic numerals (Clements & Sarama, 2009).

Counting

The only way to define larger quantities accurately is to use counting with language. In the developmental path towards successful counting, regardless of the culture, the child has to learn the number sequence, the indicating act for counting (usually pointing) and to use that indicating act to connect one number label to one entity (i.e., one-to-one correspondence), to learn methods to remember already-counted entities from as yet uncounted entities and to learn the cardinal significance of the last said number word (Fuson, 1992). Counting thus includes a knowledge of concepts (i.e., conceptual knowledge) and procedures (i.e., procedural knowledge) (Rittle-Johnson & Siegler, 1998). Conceptual knowledge refers to the understanding of counting principles (e.g., cardinality, the one-to-one principle; Gelman & Gallistel, 1978) and procedural knowledge to performing a sequence of actions in a counting task (e.g., accurately counting a set of seven objects).

Counting can be separated into *verbal* and *object counting*. Verbal counting refers to producing a sequence of number words orally or in written form by numbers. In object counting, the number of objects in a set is defined with the help of verbal counting. By the age of four most children have learnt to produce verbally the list of numbers from one to ten (Fuson, 1992) and are starting accurately to count objects of small sets using their knowledge of number words and counting principles. Besides counting arrangements of objects, children are able to produce a group of objects up to five (e.g., "Give me four sweets.") (Sarama & Clements, 2009). Gradually, as children learn to count number sequences verbally further on, they also learn to count larger collections of objects (Fuson, 1992). At the age of five, children are usually able to count backwards from ten to one both verbally and by removing objects from a group (Sarama & Clements, 2009).

At around six years, children can start verbally counting from a given number other than one (e.g., "Count from four to seven.") and can determine numbers just after or just before (e.g., "What comes just after six?") (Sarama & Clements, 2009). Gradually, children learn number sequences verbally first up to 100 and then up to 200 and even beyond, and are confident to skip count verbally (e.g., "10, 20, 30... 100") or when counting objects (e.g., "There are 2, 4, 6...12 books on the shelf."). At this age, children are beginning to understand the base-ten system (e.g., counting in units of ones, tens and hundreds) and to acquire place-value concepts (e.g., number 2 stands for a different value in 32 and in 324). Later on, children are able to count number words in both directions (forwards and backwards) and use verbal counting as a strategy in early addition and subtraction (e.g., "What is 12 more than 39? Three tens and one ten is four tens. 49, 50, 51.") (Sarama & Clements, 2009).

Comparing, ordering and estimating

Although children possess an ability to estimate and compare quantities approximately or with very small numbers (i.e., subitising) early in their life, by learning number words and counting skills they are able to *compare* the difference between two sets of quantities accurately (e.g., "How many more is eleven than eight?") (Sarama & Clements, 2009). At the age of four children start to compare small collections of objects first non-verbally by matching objects together (e.g., matching two toy cars and two marbles will result in the number of toys being the same) and then using counting (e.g., counting three sweets for both children and saying that they have the same number of sweets) (Sarama & Clements, 2009). At around the age of five, children gradually develop in making comparisons in the number range 1–10 using counting (e.g., identifying pairs of dot cards in memory games) and are later able to tell the difference between the quantities. At the age of six children are be able to know the relative size and the position of the number up to ten (e.g., "Which number is closer to five, 3 or 9?"). At the age of seven, children start to perform comparisons in the bigger number range relying on their place value understanding, and determine the relative size and position of a number using mental a number line up to 100 (Sarama & Clements, 2009).

Ordering numbers is a process, in which the child determines which of two numbers is larger than the other (Clements & Sarama, 2009). At the age of five, in addition to the cardinal aspect of a number, the children are able to identify and use the ordinal aspect of a number from the first to the tenth (e.g., "Show the third child in the line."), and later to use comparison and ordering skills together in placing collections of objects and numerals in order (e.g., putting number or dot cards in order from one to five) (Sarama & Clements, 2009).

Clements and Sarama (2009) define *estimation* as a process of solving a problem, in which a rough or tentative evaluation of a quantity is needed. Estimation can involve approximating the answer to measurement problem (e.g., about how many kilometres is it to school), to numerosity problem (e.g., how many books there are on the shelf) and to computational problem (e.g., 498 – 399) (Ramani & Siegler, 2014). After learning a number sequence, children gradually form a linear representation of numbers, a mental number line. The form of mental number line representations is often measured using a number line estimation task (Ramani & Siegler, 2014), in which children estimate a correct place for a number on a number line of arbitrary length, having ends labelled (e.g., place 32 on a number line of 0–100).

Arithmetic: addition and subtraction

When learning to count, children also start increasingly to learn arithmetic skills, to be exact, addition and subtraction. At first, children operate with small numbers using fingers and objects and different verbal counting strategies to find the answer to a problem (Fuson, 1992). Later, more developed strategies such as retrieving the answer quickly from the memory and deriving the answer through known facts (e.g., 5 + 5 = 10, so 5 + 6 is one more than ten, 11) are used (Fuson, 1992). In general, children use different strategies flexibly, according to the problem they have to solve (Sarama & Clements, 2009).

Between the age of four to five, children start to find sums for joining problems (e.g. "You have three biscuits and you get two more. How many do you have in all?") and part-part-whole problems (e.g., "You have four green balloons and one red balloon. How many in all?"), but rely on direct modelling with objects and use a counting-all strategy (e.g., counting verbally first three biscuits, then one biscuit, and then all four biscuits). Using a counting-on strategy (i.e., starting from the given number, not from one), the children are able to add objects to turn one number into another (e.g., turning three balls to five balls: counting on from three to five while putting up two more balls: "four, five"). Similarly, children are able to find the missing addend (e.g., 4 + __ = 6) by adding on objects. Furthermore, children start to know number combinations up to five (e.g., first three balls are shown, then one is secretly hidden, and then the two remaining balls are shown, the child quickly says one is hidden) (Sarama & Clements, 2009).

At the age of around five to six, instead of using a counting-all strategy, children start to prefer using a quicker counting-on strategy when finding sums for joining and part-part-whole problems (Fuson, 1992). Understanding the commutative law of addition (i.e., a + b = b + a) enables children to start counting from the bigger addend (e.g., in 2 + 9, counting forward two instead of nine), thus making counting faster. In solving missing values, such as 4 + __ = 6, children are able to count up from the first number to the target number with the help of their fingers: "five" (put up one finger), "six" (put up another finger), and resulting in two as the answer. In subtraction problems with missing values, such as 8 - __ = 6, understanding that subtraction is the inversion of addition enables children to use a counting-up-to addition strategy (e.g., “Seven, eight. From six to eight there are two steps.”) (Sarama & Clements, 2009).

At the age of six, the initial part-whole understanding has developed, and children are able to use more flexible and sophisticated strategies in solving addition and subtraction problems, some even start to use derived combinations (Sarama & Clements, 2009). The use of derived combinations is more apparent from around the age of seven years onwards, as children use known facts to solve novel arithmetical problems (e.g., using doubles, to find an answer to near doubles: 5 + 5 = 10, so 6 + 5 = 11). After the early grades, children should mostly rely on retrieving addition and subtraction facts fast from memory in the number range 1–20, and apply their place value knowledge and different strategies flexibly when solving multi-digit addition and subtraction problems (Sarama & Clements, 2009).

The mathematics content taught in the Finnish kindergarten and early grades

In Finland, participation in kindergarten education (also referred as preprimary education) is voluntary, but records show that there is almost full enrolment (Kinos & Palonen, 2013). Finnish kindergarten education is given in conjunction with public schools or day care centres.[5] There is no set time frame for the mathematics lessons that should be covered during a week. The kindergarten guidelines for the curriculum, provided by the Finnish National Board of Education, FNBE (2010b),[6] specify the aims of kindergarten mathematics in very general terms. According to the

[5] The majority (96%) of Finnish schools are public (i.e., run by municipalities). In 2013, 19.8% of kindergarten education was provided in public schools and the rest in day care centres (Official Statistics of Finland, 2013).

[6] The Finnish national core curriculum for kindergarten and basic education (grades 1-9) is being reformed, and the new curriculum will be introduced in 2016.

curriculum guidelines, children should have meaningful experiences of mathematics concepts, such as classification, seriation, comparison and quantities, mainly through play, stories, songs, physical exercise and discussions along with representational material. Kindergarten teachers are free to choose their teaching materials and methods. Several publishing houses provide kindergarten mathematics teaching materials, which are based on the objectives of the national core curriculum. Typically, these materials (e.g., Takala & Tienhaara, 2009) include topics of comparison, classification, verbal and object counting with the numbers 0–10, measurement and geometry.

The national core curriculum (NCC) provided by the FNBE (2004) specifies aims for first- and second-grade mathematics at a rather general level. The NCC includes the following sections: Numbers and Calculations (incl. number symbols, properties of numbers: comparison, classification, ordering, using concrete objects to partition numbers; addition and subtraction using natural numbers), Algebra, Geometry and Measurement. Mathematics textbooks, following the guidelines of the NCC, comprise the basic teaching material. For example, these materials for second grade (e.g., Okkonen-Sotka, Sintonen, & Uus-Leponiemi, 2009) include topics on teaching addition and subtraction skills with number symbols in the 1–1,000 range (first horizontally and then as vertical algorithms), the base-ten and place value system (1–1,000), multiplication (with numbers 1–5 and 10), introduction to division and fraction skills with manipulatives, geometry, time and measurement. In general, first and second graders have three mathematics lessons per week of 45 minutes each.

1.1.2 Individual differences in mathematics development

Sometimes a child's mathematics development does not follow the expected pace: the child does not master certain early mathematics core skills, and may show more immature strategies compared to his or her age peers. In the following, the term *low performance in mathematics* will be defined, in order to place the participants of this thesis into a precise place on a spectrum of difficulties in mathematics learning. The most severe form of difficulty, namely, *mathematics learning disability*, as well as examples of deficiencies in mathematics skills, will be presented, in order to demonstrate where the problems in the mathematics developmental path may ultimately lead.

Conceptualisations of 'low performance in mathematics' and 'mathematics learning disability'

At present, the terminology in the research literature related to describing difficulties in mathematics learning is fuzzy, as there are a variety of terms (e.g., mathematics [learning] disability, mathematics [learning] difficulty, arithmetical [learning] disability, dyscalculia), which are used inconsistently (Szűcs & Goswami, 2013). For example, studies may have used the same term (e.g., mathematics learning disability) but applied different cut-off performance criteria for defining the participants under this term (e.g., a performance in mathematics measure ranging from the lowest 35th percentile to the lowest 11th percentile) (Price & Ansari, 2013). This leads to a situation in which different levels of difficulties may be referred to by the same term in different studies. Eventually, this will hamper comparisons between the findings of different studies. To differentiate between degrees of difficulties, recent literature uses the terms mathematics learning disability and dyscalculia to describe the most severe form of difficulties (i.e., a mathematics performance score at or under the 10th percentile), and for a milder form of difficulties, the terms low performance and low achievement in mathematics (i.e., mathematics performance score between the 11th–25th percentile) are used (e.g., Desoete, Ceulemans, De Weerdt, & Pieters, 2012; Geary, 2013a, 2013b; Moeller, Fischer, Cress, & Nuerk, 2012). Furthermore, differences concerning the mathematics performance and some cognitive factors (e.g., visual-spatial ability and working memory) have been identified between these two groups (i.e., a mathematics performance score at or under the 10th percentile vs. between the 11th–25th percentile) (e.g., Mazzocco, Devlin, & McKenney, 2008; Mazzocco, Feigenson, & Halberda, 2011b; Murphy, Mazzocco, Hanich, & Early, 2007).

It has been estimated that mathematics learning disability is as common as dyslexia (i.e., developmental reading disorder) and affects 3–6% of the school-age population (Shalev, Auerbach, Manor, & Gross-Tsur, 2000). The International Classification of Diseases (ICD-10) classification system for developmental disorders (World Health Organization, 2010) refers to disabilities in mathematics as a 'mathematics disorder', and it involves "a specific impairment in arithmetical skills that is not solely explicable on the basis of general mental retardation or of inadequate schooling". This means that there has to be a discrepancy between mathematical performance and overall intelligence. It has been asserted that children with mathematics learning disability comprise a heterogeneous group, and models of subtypes of mathematics learning disability have been posited (e.g., Bartelet, Ansari, Vaessen, & Blomert, 2014; Geary 1993, 2004; Karagiannakis, Baccaglini-Frank and Papadatos, 2014;

Rubinsten & Henik, 2008). A longitudinal approach in assessment should be preferred to one based on a single point in time (Geary, 2013b; Mazzocco & Räsänen, 2013), as children may occasionally underperform in a mathematics test, leading to false positive cases of mathematics learning disability. Furthermore, Desoete, Royers, and De Clercq (2004) argue that for a child to be considered as having mathematics learning disability, in addition to the discrepancy and the severity criteria (i.e., performing two or more standard deviations below the norm in a valid mathematics test) being fulfilled, difficulties should also remain severe regardless of remedial teaching at school (i.e., resistance criterion). As children in early childhood education rarely meet the criteria for a diagnosed mathematics learning disability (e.g., the impact of formal instruction is just about to show; ICD-10, World Health Organization, 2010), in this thesis, the term low performance in mathematics is applied to children who are performing lower than expected (i.e., mathematics performance at or under the 25th percentile) compared to their age peers.[7]

In contrast to the discrepancy model, the Response-to-Intervention model (RtI) has become increasingly favoured for identifying learning disabilities, especially in the United States (L. S. Fuchs, 2003). Rather than 'wait-and-fail' (i.e., only receiving a diagnosis and starting remediation when the difficulties are already severe), the RtI model emphasises the early identification of children performing poorly in mathematics, and providing educational support immediately rather than waiting (Gresham, 2007). Evidence of disability is provided, if the child does not respond to educational support, and persists in performing below the level expected at that age (L. S. Fuchs, 2003; Gresham, 2007) (presented in more detail in chapter 1.2.3, Responsiveness to intervention). A study by Stock and colleagues (2009a) provided evidence that it is possible to identify those children who demonstrate severe difficulties in mathematics later on, already before the onset of formal schooling. In their study, 77% of the children identified as performing at or below the 10th percentile in mathematics test in kindergarten were also performing very poorly in arithmetical tasks in the first grade. A longitudinal study by Krajewski and Schneider (2009) showed that the children with low levels of performance at grade 4 were already behind in mathematics (i.e., the quantity-number concept) in the kindergarten. Furthermore, a number of other studies have shown that using validated early mathematics measures, children’s low performance in mathematics skills can be identified before the beginning of formal schooling (Aunio et al., 2006; Jordan et al., 2006;

[7] In Study I, term 'at-risk for mathematics difficulties' is used, to refer all children who perform below average mathematics skills (i.e., performing at or under the 25th percentile).

Weiland et al., 2012). However, as Morgan et al. (2009) showed, timing and persistence of low performance in kindergarten (i.e., showing low performance only in fall or spring measurement or both) can affect the mathematics growth rates from first to fifth grade. Those children who showed low performance in both times had lower growth rates compared to those who had low performance only at one measurement point.

Deficits in mathematics skills

An increasing number of studies have focused on examining nonverbal number sense, as it is suggested to lay the foundation for mathematics development (e.g., Geary, 2013a). Non-precise ANS representations, often measured with magnitude comparison tasks, has been reported to characterise school-aged children with mathematics learning disabilities (Mazzocco et al., 2011b,[8] cf.[9] Rousselle & Noël, 2007) and low-performing kindergartners (Desoete et al., 2012; Stock et al., 2009a; Toll & Van Luit, 2014). In magnitude comparison tasks, children with more precise ANS representations perform more accurately and faster, and they show smaller effects of ratio (i.e., the difficulty of the comparison is manipulated by varying the ratio of two arrays of dots, such as 10 and 5 dots or 12 and 9 dots) compared to those children with non-precise ANS (De Smedt et al., 2013).

Studies focusing on low-performing kindergartners have shown that these children often face difficulties in counting skills (e.g., Hassinger-Das, Jordan, Glutting, Irwin, & Dyson, 2014; Navarro et al., 2012; Toll & Van Luit, 2014), more specifically both in procedural and conceptual counting (e.g., Stock et al., 2009a). Difficulties in verbal and object counting may manifest themselves, for instance, in reciting number words in an incorrect sequence (e.g., missing a number word: “18, 19, 21, 22”), being unable to keep a one-to-one correspondence between counting words and objects (e.g., pointing faster to objects than saying number words), and, in general, in using more immature strategies in counting compared to those expected at that age (e.g., not being able to start counting from the given number, counting on, but instead always starting from one, counting all).

A typical characteristic of school-aged children with mathematics learning disability is that they are often unable to retrieve basic arithmetic facts from memory fast (Geary, Hamson, & Hoard, 2000; Jordan, Hanich, & Kaplan 2003; Mazzocco et al., 2008). Solving simple arithmet-

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[8] ANS precision did not differ between low- (performance between the 11–25th percentile) and typically performing students.

[9] For clarification, the meaning of 'cf.' is considered here as 'for an opposite view or a result compare to'.

ical tasks can be a difficult and slow process, as these children tend to rely on immature, slow and error-prone strategies, such as using fingers or verbal counting (Ostad, 1998).

1.1.3 Factors influencing mathematics development

A range of cross-sectional and longitudinal studies have investigated factors that could influence mathematics development among children with typical performance, low performance and with a mathematics learning disability. Typically, attention has focused on the relations of domain-general cognitive skills (e.g., general intelligence and working memory) and domain-specific skills (i.e., mathematics skills) to mathematics performance. The role of learning environmental factors in mathematics development has also been explored. Finding factors that influence mathematics development may serve as promising markers when identifying children with low performance in mathematics (Stock et al., 2009a), as well as being starting points for interventions.

Domain-general cognitive skills

Intelligence. A facility at understanding abstract information (e.g., logical and systematic relations among numbers) is the main component of intelligence (Geary, 2013a). In a study by Li and Geary (2013), intelligence measured at school-entry made an independent contribution to mathematics learning during primary school. More specifically, Hornung et al. (2014) found that after controlling for kindergarten mathematics performance, general intelligence significantly predicted first grade arithmetic and number line estimation performance. However, with children having mathematics learning disability, normal or above general intelligence has been reported (Landerl, Bevan, & Butterworth, 2004), indicating that mathematics difficulties are not originated in impairments in intelligence (Stock et al., 2009a). This is in line with the diagnosis criteria set for mathematics disorder in the ICD-10 (World Health Organization, 2010).

Working memory. Research focusing on the role of working memory (WM) in mathematics development has received increasing attention (Mazzocco & Räsänen, 2013). One of the most influential cognitive frameworks used for investigating working memory is Baddeley's multi-component model of working memory. WM is defined as a processing resource of limited capacity that allows retaining and manipulating information simultaneously (Baddeley & Logie, 1999). The model includes three components (according to Baddeley & Hitch as cited in Baddeley & Logie, 1999). The core component is *the central executive*, which refers to controlling and regulating the WM system (i.e., coordinating the two other components and allocating attentional resources to various processes).[10] The two other components, the slave systems, specialise in temporarily storing visual and spatial information (*visuospatial sketchpad*, also referred as visuospatial WM) and phonological and auditory information (*phonological loop*, also referred as verbal WM).

WM is considered one of the most important domain-general factors that influence early mathematics performance (e.g., Hornung et al., 2014; Passolunghi, Mammarella, & Altoè, 2008). Carrying out tasks of counting and mental arithmetic require moment-to-moment monitoring, processing, and maintenance of task-relevant information (Baddeley & Logie, 1999). In their recent meta-analysis, Friso-van den Bos, van der Ven, Kroesbergen, and Van Luit (2013) found that all working memory components were associated with mathematics performance in 4–12-year-old children. The findings from individual studies investigating the value of WM, and its different components for mathematics performance and development, have been inconsistent. The studies have been grounded on different theoretical frameworks, operationalised WM differently (i.e., using different types of tasks to measure each WM component, and examining only some WM components in the model), and investigated WM's relation to a variety of different mathematical skills (e.g., general mathematics tests produce stronger correlations than more skill-specific tests) (Friso-van den Bos et al., 2013). Children who perform low in mathematics (e.g., Hassinger-Das et al., 2014; Toll & Van Luit, 2014) or have a mathematics learning disability (De Weerdt, Desoete, & Roeyers, 2012; Passolunghi, 2011; cf. Landerl et al., 2004) often have problems in working memory, especially in the central executive component (Passolunghi & Siegel, 2004).

Regarding the three components of WM, Geary (2011) found that the central executive component predicted mathematics performance and performance growth from first to fifth grade. Similarly, the results of De

[10] Executive functions (EF) are generally defined as an umbrella term for higher order cognitive processes, such as planning, inhibitory control, self-monitoring, and WM (e.g., Henry, 2012, p. 21–30). However, recent conceptualisations of the central executive (e.g., Baddeley, 1996; Miyake et al., 2000) support the idea of distinct executive functions of updating, inhibition, and shifting being subcomponents of the central executive. Some researchers have included and examined these three subcomponents as part of their WM model (e.g., Friso-van den Bos et al., 2013), while some researchers have treated these three EFs separately from other WM components (e.g., Bull & Lee, 2014; Kolkman, Hoijtink, Kroesbergen, & Leseman, 2013). Regardless of the chosen viewpoint, these studies indicate that updating (i.e., in mathematics, maintaining relevant information during problem-solving, and in storing and retrieval of partial results) plays a dominant role regarding mathematics performance across a range of mathematical tasks, from early childhood to middle adolescence. Results concerning shifting (i.e., in mathematics, facilitating switching between operations, solution strategies, and the steps of a complex multistep problem) and inhibition (i.e., in mathematics, assisting in suppressing inappropriate strategies and retrieval of irrelevant associations for arithmetic facts) are less strong.

Smedt et al. (2009) showed that the central executive measured in the beginning of the first grade was a unique predictor of both first- and second-grade mathematics performance. Furthermore, Swanson (2011) found that growth on the central executive component was related to growth in word-problem-solving accuracy, from Grade 1 to Grade 3.

It has been found that the phonological loop might contribute to performance in mathematics tasks that involve language-based information processing, such as the encoding and processing of number words and numerals (e.g., Krajewski & Schneider, 2009), and retrieving linguistically stored representations of arithmetic facts from long-term memory (Östergren & Träff, 2013). More specifically, preschool and kindergarten children's verbal WM had a specific relation to mathematics skills, measured in tasks that required completing multiple steps in order to answer correctly (i.e., counting a subset, quantity comparison, and number sequences with a missing number) (Purpura & Ganley, 2014). Furthermore, the development of low-performing kindergartners' early mathematics skills during two years of kindergarten was influenced by verbal WM (Toll & Van Luit, 2014).

Visuospatial sketchpad appears to affect a number of mathematics skills (e.g., De Smedt et al., 2009), as it seems to support number representation (e.g., number inversions and reversal, alignment of column digits) and nonverbal numerical processing such as number magnitude, estimation, and representing information in a spatial form, as in a mental number line (e.g., Bull, Espy, & Wiebe, 2008). However, visuospatial sketchpad appears to relate more to early mathematics skills than to later mathematics performance, as there seems to be a shift from relying on the visuospatial system (e.g., using visual representations of quantities to solve counting and calculation problems) to an increasing reliance on the phonological system (e.g., using oral counting and fact retrieval) (e.g., De Smedt et al., 2009). Accordingly, Simmons, Singleton, and Horne (2008) found that visuospatial sketchpad functioning influenced 5-year-olds early arithmetic development (measured by orally presented simple arithmetic word problems).

Language skills. Even though approximate estimations of the quantity seems to be possible to make without language (i.e., ANS), exact representations of number are reliant on language system (Vukovic & Lesaux, 2013a). Many early mathematics tasks require using and understanding language. For instance, to count proficiently, a child needs to know number words (Cowan, Donlan, Newton, & Lloyd, 2005). For transcoding between quantities, number words and number symbols, a child has to understand the meaning of the number word and the rules that govern the structure for number words (Cowan et al., 2005). In many Western languages (for teen numbers in Finnish), this correspondence is weak, for instance, the spoken number order is the reverse of the numeral representation (e.g., "eighteen" and "18"). Being able to compare quantities involves understanding and use of linguistic concepts, such as "more" and "less". Furthermore, in mathematical story problems, a child needs to understand a range of words that can mean the same thing and can be used interchangeably (e.g., plus, and, add, together) (Purpura & Ganley, 2014). Finally, language is the core medium of teaching mathematics, and language-related mathematics concepts are frequently found in mathematics tests and instructional materials (Purpura, Hume, Sims, & Lonigan, 2011).

The nature of the interrelations between language skills and mathematics skills is not yet clear. As in the WM studies, the studies concerning language and mathematics skills may have operationalised language subskills differently (i.e., using different types of tasks to measure one skill), and investigated the relation to a variety of different mathematical skills. A growing number of international studies have shown evidence that early language skills, namely, *oral language* (i.e., vocabulary, and understanding grammatical rules and structure of language), *phonological awareness* (i.e., differentiating and manipulating meaningful segments of a spoken language, for instance, being able to blend and delete parts of words) and *print knowledge* (i.e., knowledge of letter names and sounds, words, and basic conventions about books and print), have a strong relation to early mathematics development (e.g., LeFevre et al., 2010; Praet, Titeca, Ceulemans, & Desoete, 2013; Purpura & Ganley, 2014; Purpura et al., 2011; Toll & Van Luit, 2014).

Of the various oral language skills, vocabulary knowledge has been shown to be related to overall early mathematics knowledge (Purpura et al., 2011; Purpura & Ganley, 2014) and, more specifically, to number-naming skills (LeFevre et al., 2010), in typically performing preschool and kindergarten children. Furthermore, the same studies showed that vocabulary also predicted overall mathematics performance one or two years later. A successful performance in the mathematics tasks used in the studies required a good knowledge of mathematical terms and concepts (Purpura et al., 2011). In relation to arithmetic, it seems that the more the arithmetical task includes symbolic number skills and complex arithmetical procedures the less the knowledge of vocabulary is needed, and therefore, the relation to arithmetic is weak (e.g. Simmons et al., 2008; LeFevre et al., 2010; Purpura et al., 2011; Vukovic & Lesaux, 2013a). Moreover, vocabulary and listening-comprehension skills measured together in the first grade predicted the gains from first to fourth grade in data analysis/probability, algebra, and geometry, but not in arithmetic (Vukovic & Lesaux, 2013a). However, investigating children with mathematics learning disabilities, Landerl and colleagues (2004) found that these children had normal or above normal vocabulary skills.

The relations between phonological awareness and counting skills, and phonological awareness and solving simple arithmetic calculations have been of interest, as these mathematics skills are expected to require manipulation and storage of verbal codes (Vukovic & Lesaux, 2013b). Kleemans et al. (2011) found a relation between phonological awareness and early mathematics performance (including counting skills). More specifically, phonological awareness has been related to performance in orally presented simple arithmetic problems (Simmons et al., 2008), and to arithmetic at third grade (Vukovic & Lesaux, 2013b). LeFevre et al. (2010) found that phonological awareness was related to number naming in kindergarten, and combined with vocabulary, two years later to symbolically presented numeration (i.e., quantity, order and place value), number line and calculation skills. In contrast, Passolunghi and Lanfranchi (2012) did not find a relation between phonological awareness and early mathematics performance at the end of the kindergarten year or at the end of the first grade. They argued that including more domain-general skills (e.g., processing speed and working memory), instead of only a few, into a model when predicting mathematics development reduces the influence of phonological skills on mathematics competence. Similarly, Purpura and colleagues (2011) found that, although phonological awareness was correlated with later mathematics performance, the predictive relation to mathematics performance was fully accounted for by vocabulary and print knowledge. In addition, LeFevre and colleagues (2010) found no relation between phonological awareness and arithmetical problems which did not require using language, a result which is similar to Cirino's (2011), who found only an indirect effect (through symbolic labelling) on addition problems.

Relation between print knowledge and mathematics performance is not well studied (Purpura et al., 2011). However, a study of Purpura et al. (2011) found that print knowledge (measured as print concepts, letter discrimination, word discrimination, letter name identification, and letter sound identification) measured in preschool predicted mathematics performance (except for arithmetic) one year later. The mathematics tasks used in Purpura et al.'s (2011) study required children to rely on printed numbers, mathematical symbols (e.g., +, -), and the meanings behind this written symbolism. In contrast to Purpura et al.'s (2011) finding that print knowledge of preschoolers did not predict arithmetic skills one year later, Zhang et al. (2013) found that letter knowledge measured in kindergarten predicted the level of arithmetic (addition and subtraction) performance in the first grade, and later progress through the third grade, above and beyond the effects of phonological awareness and receptive vocabulary.

Investigating children with a specific language impairment (SLI) has provided further evidence of the role of language in the development of mathematics skills.[11] Children with SLI often show significant limitations in WM, more specifically in the central executive and phonological loop (Montgomery, Magimairaj, & Finney, 2010). Interpretation of the results is hence made complex, if WM has not been included in the studies as one variable. Owing to difficulties in learning to recite number sequences correctly, children with SLI have been found overall to show weaker counting skills (Cowan et al., 2005; Fazio, 1994, 1996; Kleemans, Segers, & Verhoeven, 2011) and arithmetic skills (Donlan, Cowan, Newton, & Lloyd, 2007; Koponen, Mononen, Räsänen, & Ahonen, 2006) compared to their age peers. However, children with SLI often seem to understand conceptual counting principles (e.g., cardinality and one-to-one correspondence) (Arvedson, 2002; Fazio, 1994, 1996) and arithmetical principles. As comparing, ordering and estimating skills seem to require less language processing compared, for instance, to counting, children with SLI have given a similar performance to their age peers, and above their language peers, in skills such as single-digit number comparison (Donlan, Bishop, & Hitch, 1998; Donlan & Gourlay, 1999) and number line estimation (Kleemans et al., 2011).

Domain-specific mathematics skills

The influence of domain-specific mathematics skills on mathematics development has been investigated both in cross-sectional and longitudinal studies. In these studies, mathematics skills have been treated either in terms of general mathematics competence (based on a composite score of the test) or in terms of as a specific mathematics skill (e.g., counting). Both types have been used as either a dependent or independent variable, thus making the outcomes of the studies varied. Furthermore, some studies have controlled the effect of domain-general cognitive skills (cognitive skills included in the studies have varied across studies), whereas some have not.

In general, the influence of early mathematics skills seems to be strong in predicting mathematics development (e.g., Duncan et al., 2007; Hornung et al., 2014; Jordan, Glutting, & Ramineni, 2010; Jordan, Kaplan,

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[11] In SLI, the normal language acquisition patterns are disturbed in the early stages of development, conditions not caused by neurological or speech mechanism abnormalities, sensory impairments, mental retardation, or environmental factors (ICD-10: World Health Organization, 2010).

Locuniak, & Ramineni, 2007; Krajewski & Schneider, 2009). Those children who begin with good mathematics skills also appear to perform well later on (Aubrey et al., 2006), but children with weaker skills often remain low-performing throughout their school career (Duncan et al., 2007; Morgan, Farkas, & Wu, 2009), and the gap between them and typically performing children has even been shown to widen during their school years (Aunola et al., 2004).

From the mathematics learning disability point of view, there are two mathematics skills that are of most interest; nonverbal number sense and arithmetic skills. Accordingly, the influence of nonverbal number sense on other mathematics skills is appealing, as it has been suggested to be the foundation for mathematics development, and weak ANS precision has been found to characterise school-aged children with mathematics learning disabilities (Mazzocco et al., 2011b). Second, mathematics skills related to arithmetic skills are of interest, as school-aged children with mathematics learning disabilities often have severe problems in arithmetic skills.

A relation between nonverbal number sense (ANS) and overall mathematics competence was found in preschoolers (Bonny & Lourenco, 2013) and in kindergartners, even when WM was taken into account (Hornung et al., 2014). More specifically, ANS was found to influence counting and arithmetic skills via symbolic approximation skills (Xenidou-Dervou, De Smedt, van der Schoot, & Lieshout, 2013). Mazzocco and colleagues (2011a) provided evidence that ANS precision, measured in preschool, predicted mathematics performance two years later. Kroesbergen et al. (2009) discovered that subitising skills were related to kindergartners' counting skills (cf. Soltész, D. Szücks, & Szücks, 2010) and explained that this relation was found because linking the number words to quantity representations is relevant in counting. Furthermore, non-symbolic magnitude comparison, measured in kindergarten, was found to predict arithmetic skills in first and second grade (Desoete et al., 2012; cf. Bartelet, Vaessen, Blomert, & Ansari, 2014).

There is general agreement that the role of counting skills is a significant predictor of early grades arithmetic skills, based on findings from several longitudinal studies (e.g., Aunio & Niemivirta, 2010; Aunola et al., 2004; Bartelet, Vaessen, et al., 2014; Desoete et al., 2009; Lepola et al., 2005; Stock et al., 2009a, 2009b). In studies of Aunola et al. (2004) and Lepola et al. (2005), counting skills were measured as verbal number-word sequence skills, whereas in other studies counting also included enumeration tasks (e.g., counting how many dots there are). Performance in later arithmetic skills has also been predicted by using a composite score from tests of kindergarten early mathematics skills: the better the early mathematics skills, the better the performance one to three years later in arithmetic skills (Locuniak & Jordan, 2008; Östergren & Träff, 2013).

Learning environmental factors

Children come from families of dissimilar socio-economic background. Studies that have investigated and compared the early mathematics performance between children from low-income and middle-income families have revealed that children from low-income families often lag behind and make less progress in early mathematics skills than their peers (e.g., Jordan et al., 2006, 2007; Siegler & Ramani, 2008; cf. Aunio & Niemivirta, 2010, in which the socio-economic status had a significant effect on only few early mathematics skills in Finnish children). Compared to children from middle-income families, children from low-income families have had less explicit opportunities and support from their home environment for learning foundational early mathematics skills (Siegler, 2009), likely influenced by financial constraints and parents' lower education in low-income families (Ramani & Siegler, 2014). If adequate early support is not provided for children from low-income families, there is a risk that these children will also exhibit low performance later on at school, as their foundational early mathematics skills are not strong enough to support the learning of more advanced mathematics at school.

To sum up, both domain-general and domain-specific factors, as well as, learning environment seem to influence the development of mathematics skills. Whether some specific factor has an independent contribution to mathematics performance and development or not, appears to depend on a variety of factors used together in the studies (e.g., general intelligence, some components of working memory, some mathematics skills, socio-economic status) and measures used for assessing these factors, thus possibly resulting in an inconsistency in such findings (Hornung et al., 2014). Regarding the children having difficulties in mathematics, different sample selection criteria (i.e., cut-off score) used in the studies may also have resulted in inconsistent findings.

1.2 Supporting early mathematics development

As there is a wide variation in children's early mathematics performance even before formal schooling (e.g., Aubrey et al., 2006; Aunio et al., 2006; Aunio & Niemivirta, 2010; Stock et al., 2009a) and early mathematics skills seem to be a strong predictor of mathematics development (e.g., Duncan et al., 2007; Hornung et al., 2014; Jordan et al., 2007, 2010; Krajewski & Schneider, 2009), legal and practical actions have been made to develop educational support for all children, in order to meet the diversity in children's learning support needs (Finland's Basic Education Act 628/1998, Amendment 642/2010; Lembke et al., 2012). In this thesis, the focus is on educational support in the form of mathematics interventions.

1.2.1 Conceptualisation of evidence-based intervention

In this thesis *intervention* is defined according to Tilly and Flugum (as cited in Riley-Tillman & Burns, 2009, p. 2) as "a planned modification of the environment made for the purpose of altering behavior in a prespecified way".[12] In the context of early mathematics interventions, this means implementing a specific mathematics programme for a specified group, in a specified time frame and of a certain intensity, in order to improve mathematics performance. There have been growing demands to provide educators with interventions that have proved their functionality and effectiveness in improving learning outcomes. A specific criterion for determining whether an intervention is *evidence-based* is currently unavailable (Kratochwill, Clements, & Kalymon, 2007), but certain generally accepted principles can be outlined. An intervention can be considered as evidence-based if it has undergone a well-designed and robustly implemented experimental study, or preferably several studies (Brown-Chidsey & Steege, 2005). This means conducting a systematic experiment with methods that have proved valid and reliable data, a thorough data analysis, and a detailed description of the participants, setting and methodology (e.g., Forbringer & W. W. Fuchs, 2014, p. 2; Gersten et al., 2005; Slavin, 2008). Furthermore, a study has to be accepted by an objective review such as a peer-reviewed journal. Based on the evidence gathered from the studies concerning one particular intervention, its effectiveness can be rated (e.g., positive, mixed or negative), in order to guide educators in choosing the most effective interventions for their use (What Works Clearinghouse, 2008). In implementing evidence-based interventions that are effective with integrity,[13] the expectation is that it will increase the probability of positive learning outcomes for students (Brown-Chidsey & Steege, 2005, p. 30).

A recent review by Kroeger, Douglas Brown, and O'Brien (2012) showed that of twenty commercially available mathematics interventions for children from kindergarten to grade three, only a quarter of them met

[12] In the original studies, the terms intervention programme or instruction are also used but with a meaning similar to the term intervention.

[13] Integrity refers to the degree to which an intervention is implemented as designed (Brown-Chidsey & Steege, 2005, p. 32)

the criterion of being evidence-based. Developing a novel intervention based on a theoretical background, conducting studies examining the intervention, publishing the results and making the intervention available for educators may take several years in total. Therefore evidence-based interventions are often slow to become part of the instruction in kindergartens and schools. The demand that educational practices (e.g., interventions) fulfil strict evidence-based criteria has been criticized, and a more liberal approach has also been proposed in response, namely, *evidence-informed practice* (Hammersley, 2013, pp. 8, 38–39). Rather than implementing evidence-based interventions in the same manner that researchers do, there has been an increasing emphasis on the idea that educators interpret and evaluate the evidence, and combine it with other situation specific information when making judgements regarding what is the best practice in a particular situation (Hammersley, 2013, pp. 38–39).

1.2.2 A three-tiered model in Finnish educational support

Organising educational support in Finnish kindergartens and basic education (grades 1–9) is based on three tiers: general, intensified and special support (Finland's Basic Education Act 628/1998, Amendment 642/2010; FNBE, 2010a), and it shares similarities with the Response to Intervention (RtI) approach used in the United States (Lembke et al., 2012). The focus in the three-tiered model is on providing support as early as possible in order to prevent the emergence and growth of difficulties (FNBE, 2010a). The child can move between the three tiers according to the need for support (FNBE, 2010a).

In general support (Tier 1), the focus is on providing quality core instruction and identifying potential learning problems of all the children. Researchers have suggested that with Tier 1 instruction the needs of around 80% of children can be met (Batsche et al., as cited in Peterson, Prasse, Shinn, & Swerdlik, 2007) (Figure 1). The quality of instruction can partly be ensured by using such core curricular programmes with integrity that have been shown to improve performance (Lembke et al., 2012). At Tier 1, differentiation of instruction is a primary means of taking the diversity of students into account, and it may focus on areas such as subject content, teaching materials, the methods applied and the amount of school- and homework (FNBE, 2010a). Short term extra tutoring, a form of differentiation, can be provided as a preventive means during or outside mathematics lessons. At Tier 1, children's performance in mathematics should be assessed on a regular basis, using screening measures in order to identify those children who perform less well compared to the expected age performance level. For those children who have been identified with low levels of performance, further assessment should be conducted in order to specify in more detail the areas of mathematics where there are difficulties. In this thesis, Study I reviewed intervention studies that had used core curricular programmes, and aimed to show their effectiveness and pedagogical components. At present such validated core curricular programmes do not exist in Finland. Therefore, Study II investigated the effects of RightStart Mathematics core curricular programme in kindergarten classes within general support. Of interest was also, whether the programme and general support was effective enough to support the children initially performing low in early mathematics skills.

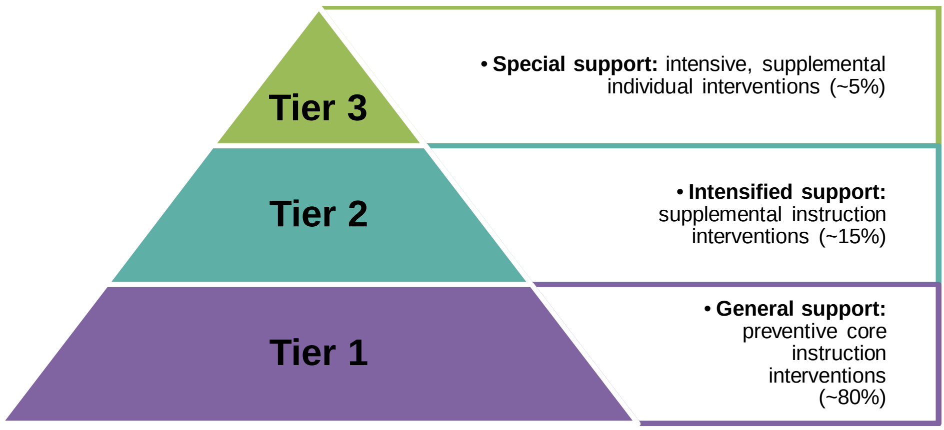

Figure 1. A three-tiered model of educational support and intervention (adapted from Batsche et al., as cited in Peterson, Prasse, Shinn, & Swerdlik, 2007, p. 305).

If the child does not respond sufficiently to Tier 1 instruction, the child's progress in learning is assessed and handled by a multi-professional team at the kindergarten or school (FNBE, 2010a). If intensified support is needed, a learning plan is made for the child. At Tier 2, usually comprising around 15% of children,[14] part-time special education has a significant role in supporting children who need intensified mathematics instruction (FNBE, 2010a). This instruction is typically organised as pull-out lessons during mathematics lessons or in the form of co-teaching where a special

[14] In 2013, in Finland, 6.5% of comprehensive school (grades 1-9) students received intensified support and 7.3% special support (Official Statistics of Finland, 2013). Looking at the actual percentages in relation to what are considered to be ideal percentages, presented in Figure 2, it must be taken into consideration that the current support system has been in use only from the beginning of 2011.

education teacher works together with the teacher in the classroom. As part of the intensified support, children can also receive systematic extra tutoring. In contrast to the RtI model (D. Fuchs & Fuchs, 2001), no specific guidelines about the length or the intensity of Tier 2 instruction is given in the Finnish model. Furthermore, the RtI model emphasises that at Tier 2, supplemental intervention programmes that have shown evidence of improving learning should be used with integrity. Currently, in Finland, there are only a few commercially available research-based mathematics supplemental intervention programmes for supporting young children performing low in mathematics; namely, for 3–5-year-old children NalleMatikka [Teddybear Math] (Mattinen, Räsänen, Hannula, & Lehtinen, 2010); for 4–7-year-old children Minäkin Lasken! [I Count, too!] (Van Luit, Aunio, & Räsänen, 2010) and the computer programmes Graphogame Math and Number Race (Lukimat-webservice) for kindergartners and first graders. In this thesis, Study I reviewed interventions that targeted children who needed supplemental support in their mathematics learning. Furthermore, Study IV investigated the effects of intensified mathematics instruction for second graders performing low in mathematics. For this purpose, a new supplemental intervention programme, the Improving Mathematics Skills in the Second Grade (Mononen & Aunio, 2012), was developed.

If the child does not respond to the instruction at Tier 2 or performs poorly in subsequent assessments, a more extensive pedagogical assessment of the child's progress in learning is conducted (FNBE, 2010a). Based on this information, an official decision concerning special support (Tier 3 instruction, usually comprising around 5% of students) can be made by the school administrator (e.g., the head master), and an individual education plan, IEP (i.e., a written plan relating to the child's learning and covering educational content, pedagogical methods and other necessary support services), is drawn up for the child (FNBE, 2010a). This decision on special support must be revised at least after the second grade and prior to moving on to the seventh grade (Finland's Basic Education Act 628/1998, Amendment 642/2010). In the case of mathematics, at Tier 3 the child studies in accordance with an individualised syllabus in mathematics (instead of the general syllabus), and the child's performance is assessed on the basis of the individual objectives set out for the child in the IEP. The organisation of Tier 3 instruction shares similarities with that of Tier 2 instruction. Compared to Tier 2 instruction, instruction at Tier 3 is more intensive and is targeted in more depth at the individual needs of the child. In this thesis, Study III investigated the effect of RightStart Mathematics programme in the sample of children with SLI, who attended a special education school for children with SLI (Tier 3 support) and who had an IEP in mathematics.

1.2.3 Responsiveness to intervention

An adequate or inadequate response to intervention serves as a decision-making tool in guiding further actions in the three-tiered model of support, such as providing more intensive instruction (Gresham, 2007). According to the literature, there does not seem to exist any exact method for determining what constitutes an adequate response to intervention (Gresham, 2007). Regarding the area of academic performance, L. S. Fuchs (2003) has proposed two approaches for determining the child’s response to intervention: (1) final status performance and (2) growth models. Final status refers to the performance of the child at the end of the intervention based on a normative or a criterion-referenced benchmark. The response to intervention based on a final status performance may be considered adequate, if the child performs in the normative range on a norm-referenced measure of mathematics (e.g., above the 25th percentile) or exceeds established benchmark criteria for a particular mathematics skill (e.g., a first grader calculates 20 addition facts in two minutes). However, the child can make excellent progress without necessarily achieving the final normative or benchmark criterion (L. S. Fuchs, 2003). Therefore, the child’s progress (i.e., growth) should also be considered by comparing the child’s performance before and after the intervention. Figure 2 depicts three hypothetical growth models that might be expected in response to a mathematics intervention, originally introduced as hypothetical responses to reading interventions by Gresham (2007).

The purple line in figure 3 represents the typical growth over time in children with average performance in mathematics. Hypothetical group A represents those children who, at the beginning, have a lower level of performance compared to their average performing peers, but, when provided with intervention, catch-up with their peers. Such children might be considered as 'instructional causalities' (Gresham, 2007); they are not true cases with mathematics learning disability, instead the causes for their initial low level of performance might have been poor and inadequate instruction or the lack of meaningful early mathematics experiences (e.g., as in children coming from low socio-economic families). Hypothetical group B begins behind their average performing peers, but progress at the same rate. Regardless of their steady improvement this group is not able to close the performance gap with their peers, and thus does not reach the desired level of performance in mathematics. Hypothetical group C exhibits pattern of inadequate response to intervention (i.e., treatment resistance). These non-responders' (Gresham, 2007) initial performance level is behind their average performing peers and despite intervention they continue to fall further behind their average performing peers. According to L. S. Fuchs & Fuchs (as cited in L. S. Fuchs, 2003), in the RtI model, only this group of children, who manifest severe discrepancies from their peers both in terms of growth and in terms of level (i.e., dual discrepancy), would be considered as having mathematics learning disability. Clearly, the duration of intervention should be taken into consideration when considering the children's responsiveness to intervention. For example, when providing a short intervention (e.g., few weeks or months), it cannot be concluded that the non-responder has mathematics learning disability, rather a longer period of evidence of non-responsiveness is needed.

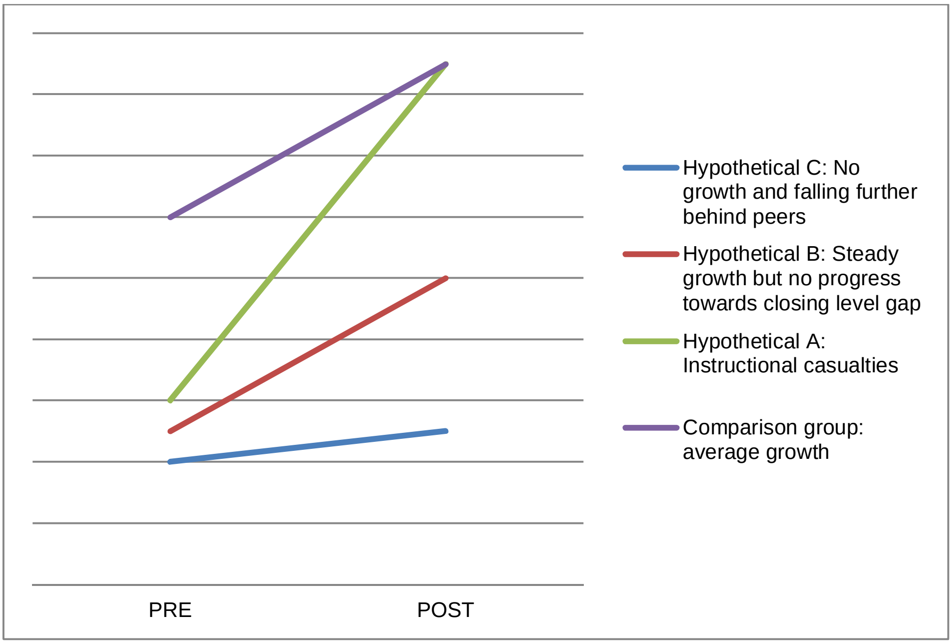

Figure 2. Hypothetical responses to mathematics intervention (adapted from Gresham, 2007, p. 19, originally describing responses to reading interventions).

1.2.4 Key findings from mathematics intervention review studies

To give an overview of what kind of information mathematics intervention studies have provided in general, findings from thirteen systematic reviews and meta-analyses (hereafter referred to as reviews) are presented. These intervention reviews concern school-aged children performing low in mathematics or having learning disabilities (i.e., having significant difficulties in the acquisition and use of listening, speaking, reading, writing, reasoning or mathematical abilities; Hammill, Leigh, McNutt, & Larsen, 1988). The reviews are approached here in terms of: (1) the potential for increasing mathematics performance, (2) the essential instructional features to promote learning and (3) the methodological factors related to the intervention's effectiveness. A summary of the review studies is presented in the Appendix. In addition to providing information for educators about the most effective interventions for students with low performance in mathematics or with learning disabilities, the key findings from these reviews can provide guidelines for researchers in developing and examining novel interventions. As the focus of this thesis is on children with low performance in mathematics, the reviews examining mathematics instruction for typically developing children (Slavin & Lake, 2008) or for more specific disability subgroups, such as children with emotional and behavioural disorders (Templeton, Neel, & Blood, 2008), are not addressed here.

Increase in mathematics performance

In a majority of the reviews, the effects of interventions focusing on *computation* (i.e., basic arithmetic skills of addition, subtraction, multiplication and division) were examined (e.g., Codding, Burns, & Lukito, 2011; Codding, Hilt-Panahon, Panahon, & Benson, 2009; Gersten, Chard, et al., 2009; Kroesbergen & Van Luit, 2003; Kunsch et al., 2007, Methe, Kilgus, Neiman, & Riley-Tillman, 2012). Interventions that targeted improving computation skills were effective: the degree of effectiveness ranged from moderate to high. Across all grades, interventions focusing on *word-problem-solving* skills have received attention (e.g., Kroesbergen & Van Luit, 2003; Maccini, Mulcahy, & Wilson, 2007; Xin & Jitendra, 1999; Zhang & Xin, 2012). Besides weak arithmetic skills, word-problem-solving appears to be one of the areas that children with low performance in mathematics often confront (e.g., Geary, 1994). These interventions were beneficial in boosting word-problem-solving skills, exhibiting a moderate to high degree of effectiveness. Zhang and Xin (2012) revealed that students with low performance in mathematics and with learning disabilities responded similarly to word-problem-solving interventions. Typically, however, the reviews included interventions for *a range of mathematics skills* (e.g., Baker, Gersten, & Lee, 2002; Fischer, Moeller, Cress, & Nuerk, 2013; Gersten, Chard, et al., 2009; Miller, Butler, & Lee, 1998), for instance, when reviewing all mathematics interventions targeted at school-aged students with learning disabilities (e.g., Gersten, Chard, et al., 2009). Some researchers further categorised and analysed the interventions in their reviews according to the mathematics area, thus enabling one to compare the different skill areas (e.g., Kroesbergen & Van Luit, 2003). Interventions targeting *early mathematics skills* were the main focus in only one review (Malofeeva, 2005). Although early mathematics instruction proved to be effective, Malofeeva (2005) did not analyse the effect in terms of the type of participants involved (interventions included typically and low-performing children). Furthermore, Kroesbergen and Van Luit (2003) found a high degree of effectiveness in interventions in preparatory skills (i.e., early mathematics skills) for children with special educational needs in mathematics. In sum, mathematics interventions have generally proved to be moderately to highly effective in increasing the mathematics performance of students with a low level of performance in mathematics or having learning disabilities.

Essential instructional features to promote learning

Five instructional features exhibited a moderate to high degree of effectiveness in several reviews: (1) explicit instruction, (2) peer-assisted instruction, (3) computer-assisted instruction, (4) self-instruction and (5) applying a concrete-representational-abstract sequence (CRA). These instructional features could thus be considered as essential in mathematics interventions, at least for school-aged children performing low in mathematics or having a mathematics disability. A more detailed description of the effective features found in each review study is presented in the Appendix.

Explicit instruction (also referred to direct instruction) includes elements of modelling mathematics concepts and strategies for students step-by-step, guided and independent practice opportunities and continuous feedback (Forbringer & W.W. Fuchs, 2014). It was found to be an effective instructional feature for students performing low in mathematics (e.g., Baker et al., 2002) or with learning disabilities (e.g., Gersten, Chard, et al., 2009; Kroesbergen & Van Luit, 2003; Miller et al., 1998). Explicit instruction was also incorporated into other instructional features such as in using problem structure representation (Zhang & Xin, 2012) or in strategy training (Xin & Jitendra, 1999) in word-problem-solving tasks. When combined with guided teaching it was found to be the most beneficial instructional feature in teaching early mathematics skills (Malofeeva, 2005). More specifically, explicit instruction was found to be the most effective instructional feature in teaching computation skills for primary school aged children with special educational needs (Kroesbergen & Van Luit, 2003).

Peer-assisted instruction (also referred to peer-mediated instruction or peer tutoring) involves pairs of students working collaboratively on structured, individualised activities, and enables peers to provide an answer, or provide suggestions that help them solve the problem (Baker et al., 2002; Kunsch et al., 2007). Peer-assisted instruction was an effective instructional feature especially with primary school aged children performing low in mathematics (Baker et al., 2002; Kunsch et al., 2007). The effect was least noticeable when the instruction was used with secondary school aged students or with children with learning disabilities (Kunsch et al., 2007). Kroesbergen and Van Luit (2003) did not find a statistically significant effect for peer-assisted instruction for children with special educational needs. Kunsch et al. (2007) explained that the finding that peer-assisted instruction is not such an effective feature among children with learning disabilities may result from the fact that there is a likelihood that both tutors and tutees are cognitively disadvantaged for the required actions in peer-assisted instruction.

In *computer-assisted instruction* (CAI), technology is utilised, such as in using a mathematics computer software. CAI was an effective way to improve performance in word-problem-solving skills with children with low performance in mathematics or with learning disabilities in both primary and secondary school (Xin & Jitendra, 1999). Reviews by Miller et al. (1998) and Kroesbergen and Van Luit (2003) also supported using CAI with children with learning disabilities or with special educational needs, although Kroesbergen and Van Luit (2003) reported that interventions using CAI had less of an effect than interventions in which a teacher instructed the students.

Self-instruction (also referred to self-management and self-monitoring) refers to providing students with a set of verbal cues (e.g., checklists) as mediators for cognitive and metacognitive operations so they can remember what they are doing (Goldman, 1989). Overall, self-instruction was an effective instructional feature for primary school children with special educational needs in mathematics (Kroesbergen & Van Luit, 2003) and was a beneficial instructional feature in teaching computation skills (e.g., Codding et al., 2009, 2011; Miller et al., 1998) for students performing low in mathematics and with learning disabilities.

The concrete-representational-abstract sequence (CRA) is a three-phase process that involves first instruction using concrete objects (e.g., mathematics manipulatives), gradually advancing to pictures that represent objects and finally to abstract level using numbers and symbols (Maccini et al., 2007). CRA was reported to be an effective instructional feature especially in teaching computation skills (Maccini et al., 2007; Methe et al., 2012). Furthermore, using manipulatives and drawings in teaching a range of mathematics topics (Miller et al., 1998) and more specifically in teaching word-problem-solving (Xin & Jitendra, 1998) proved to be an effective instructional feature for students performing low in mathematics or with learning disabilities.

Methodological factors related to an intervention's effectiveness

Comparing the effectiveness of different interventions is not straightforward, as there are a number of issues that have been found to relate to the size of the effect. The review by Fischer et al. (2013) focused merely on the evaluation of the impact of methodological factors in intervention studies related to their effectiveness. Several other authors of reviews also considered the topic as part of their research (e.g., Codding et al., 2011; Gersten, Chard, et al., 2009; Kroesbergen & Van Luit, 2003; Kunsch et al., 2007).

First, *the measurement* may mediate the effectiveness of the intervention. The effects of interventions that have used standardised tests have been shown to be significantly lower than in interventions that have used self-developed tests (Zhang & Xin, 2012). Self-developed tests often specifically measure the skills practised in the interventions, therefore producing larger effects. In contrast, in the review of Kunsch et al. (2007), the type of measurement did not moderate outcome, as relatively long interventions (from 10 weeks to almost one year), according to the authors, may have masked the effects for the type of measurements.

Second, the type of *control group* may affect the size of the effect. Studies that provided another type of intervention for the control group (i.e., active control group) tended to have a significantly smaller effect when compared to studies that did not (i.e., using passive control groups) (Fischer et al., 2013; Gersten, Chard, et al., 2009). Furthermore, if the study employed a performance-matched control group, it produced smaller effects than those that did not (Fischer et al., 2013).

Third, because of the different nature of *group- and single-subject design* interventions (e.g., single-subject designs often include baseline measurements and no control participants), to get reliable effect size values, effect size calculations should be performed differently and the results should be treated separately for group- and single-subject design studies. Consequently, some of the reviews (e.g., Xin & Jitendra, 1999; Zhang & Xin, 2012) handled group- and single-subject design studies separately; some decided not to include single-subject designs in their reviews (e.g., Malofeeva, 2005; Kunsch et al., 2007). Kroesbergen and Van Luit's review (2003) was criticized (Gersten, Chard, et al., 2009) for producing some bias for effect sizes, as they did not analyse the different designs separately. However, Kroesbergen and Van Luit (2003) reported that single-subject design studies had significantly higher effect sizes than the group design studies. Also, small studies (in terms of the number of participants) presented higher effects than large studies (Kroesbergen & Van Luit, 2003).

Fourth, Fischer et al. (2013) stated that *the number of mathematics components* in an intervention is related to the size of the effect. They found that interventions that focused on training one specific mathematics skill (i.e., one component) produced significantly greater effects than interventions that covered more than one mathematical skill (i.e., multicomponential).

Fifth, interventions that focus on a specific skill are often of *short duration.* As a specific mathematics skill can often be fully acquired in a short period of time (Kroesbergen & Van Luit, 2003), shorter interventions have generally found to be more effective than longer ones (Gersten, Chard, et al., 2009; Kroesbergen & Van Luit, 2003). In contrast to this finding, the length of an intervention was not found to affect intervention outcomes in reviews by Kunsch et al. (2007), Fischer et al. (2013) and Malofeeva (2005). These contradictory results may be explained by the fact that the review studies involved defined the length of a long and short intervention differently. Moreover, the length of an intervention does not tell us how intensive the intervention is (i.e., duration and frequency of sessions).

Sixth, *the type of interventionist* (i.e., a researcher or a teacher) has, on the one hand, been shown to have had a minimal impact on intervention outcomes (Gersten, Chard, et al., 2009), and, on the other hand, Codding et al. (2011) found that using a combination of 'intervention agents' (i.e., a teacher and a student or a teacher and a researcher) was the most effective approach.

Finally, there are contradictory findings whether interventions are more effective for *younger or older students*. Reviews by Gersten, Chard, et al. (2009), Kunsch et al. (2007) and Methe et al. (2012) found larger effects with younger students. In contrast, Kroesbergen and Van Luit's review (2003) found more of an effect with older students than with younger students, and in Xin and Jitendra's review (2012) the age of the students did not affect the intervention's effectiveness. In interpreting these results, one should note that the range of ages or the school level varied in different reviews and some age levels may have been overrepresented. For instance, the review by Kunsch et al. (2007) included students from primary to secondary school, and Kroesbergen and Van Luit's review (2003) included students from kindergarten to primary school. Xin and Jitendra's review (2012), however, included students from primary to post-secondary school (elementary 34%, secondary 49% and post-secondary 17%).

1.2.5 Mathematics intervention programmes in this thesis

In this thesis, two programmes were used in the intervention studies: one aimed at core instruction, The RightStart Mathematics (RS) programme (Cotter, 2001) and one for intensified instruction, Improving Mathematics Skills in the Second Grade (IMS-2) (Mononen & Aunio, 2012). The RS programme was chosen as it was research-based (Cotter, 1996) and included features not found in typical Finnish kindergarten mathematics programmes, such as employing transparent number naming, emphasising non-counting strategies and systematic use of manipulatives. The IMS-2 programme was chosen as I was involved in the development and research work of this programme in the ThinkMath project. Neither of the programmes had previously been investigated in Finland.

RightStart Mathematics Kindergarten

The RS kindergarten core curriculum programme (Cotter, 2001) is the outcome of Cotter's dissertation work (1996). Today, the programme has materials covering all the primary grades (www.rightstartmath.com). In the kindergarten programme, learning to name numbers is based on the transparent base-10 number-naming system (e.g., 14 is 'ten-four', 23 is 'two-ten three'), and then followed by learning the English number-naming system. The transparent number-naming system used in the RS programme has been shown to have a positive effect on the learning of mathematics skills, compared to the Western irregular number-naming system (Miura & Okamoto, 2003). Second, the programme emphasises non-counting strategies in object counting. Subitising skills are encouraged in counting small quantities (1–4) by saying the total quantity instead of counting one by one. In addition, groupings of fives and tens are used. For example, the number seven is first taught as 'five and two' and demonstrated with beads of two different colours on an abacus (e.g., five blue and two yellow beads), or 9 + 4 is calculated by changing the amount to 10 + 3 on an abacus. For the most part, the RS programme focuses on manipulating numbers between 0 to 20. However, children are introduced to numbers up to 1,000 (i.e., place value knowledge), too, with supporting manipulatives, as well as calculations with tens and ones with the help of an abacus (e.g., 30 + 30, 44 + 1 or 57 + 2).

The CRA sequence is applied in the instruction. A new concept is practised with a concrete manipulative (e.g., showing a quantity of five with tally sticks or on an abacus), then followed by representational material (e.g., quantity of five as tally marks on a card), and finally practised as an abstract representation (e.g., symbol of the number five on a card). Written work with numbers (i.e., worksheets) is postponed until a child has understood the mathematic concept. In these activities, all the children have access to manipulatives: Abacuses based on groupings of five and ten beads with two colours (also known as Slavonic abacuses), number and quantity cards, base-ten cards, tiles and tally sticks are regularly used throughout the programme. A teacher has manipulatives similar to those of the children, but on a larger scale (e.g., a large abacus) to aid teaching.

There are 77 lesson plans in the RS kindergarten manual for the kindergarten year. One lesson is composed of a short warm-up activity (usually practising different types of number word sequences, subitising, or days of the week) and from three to six activities (e.g., teacher-guided or pair activities with manipulatives or card games) around one or two learning objectives. In learning, understanding is highly emphasised, not learning by rote. The role of the teacher is to encourage thinking by asking questions and having discussions with children, not only giving answers. The activities are supposed to be done following the order given in the manual, as one activity may require skills taught in the previous activity. Instructions for the activities are specific, including questions to be asked by the teacher.

In comparing the learning objectives of the RS programme with the Finnish national mathematics core curriculum guidelines, it was found that the RS covered the main learning aims and was therefore eligible for use with Finnish kindergartners. The author of the programme, J. A. Cotter, gave permission to translate the original material into Finnish and to use the programme for research purposes. The Finnish version includes 87 lessons; for practical reasons, some of the two-hour lessons were divided into two one-hour lessons. Furthermore, some cultural aspects affected the translation (e.g., the money used is the euro instead of the dollar, a 24-hour clock is used instead of a 12-hour clock), but the content of the manual and the tasks were kept as similar to the original manual as possible.

Improving Mathematics Skills in the Second Grade

Improving Mathematics Skills in the Second Grade (IMS-2) programme focuses on practising counting skills and conceptual place value knowledge in the 1–1,000 number range. When the content of the programme was designed, the development of early mathematics skills was taken into consideration. Furthermore, instructional features that were found to have been effective with children performing low in mathematics were considered. The IMS-2 intervention is designed to be implemented with a small group of children (maximum of six), as a supplemental instruction for core mathematics instruction. The teacher manual includes 12 lesson plans of 35–45 minutes each. The first three sessions include working with tens (e.g., ten ones equal one ten, ten tens equal one hundred, adding and subtracting tens with objects, and skip counting by tens). The next three sessions include practising relations between of number words, numbers and quantities in the number range 10–100. The following five sessions include practising in the number range 1–1,000, so that children will work first with hundreds, then with hundreds and tens, and finally with hundreds, tens and ones. In these five sessions the relations between of number words, numbers and quantities are practised together with number word sequence skills. The last session is a snake-and-ladders game with revision questions.

The programme applies guidelines of explicit teaching and the CRA sequence. Each lesson consists of a teacher-guided activity to model a new mathematical learning concept or strategy, as well as guided and peer activities (e.g., hands-on activities with manipulatives or card and board games for the current topic), and, at the end of the lesson, there is a short, paper-and-pencil individual activity. Mathematical ideas are represented following the CRA sequence, thus giving meaning to abstract concepts using visual representations (e.g., cubes and bundles of sticks). Structuring numbers (e.g., Ellemor-Collins & Wright, 2009) is one of the key elements in representing quantities (i.e., using dot cards structured in tens and hundreds). Structuring numbers is rarely used systematically in Finnish mathematics teaching materials. The lesson plans include specific instructions for teachers to follow in each activity. The manipulatives are made of low-cost, everyday materials to be found in every classroom, combined with printable materials (e.g., dot and place value cards) included in the manual.

1.3 The present study

1.3.1 Aims

The development of early mathematics skills is a well-researched area, and the influence of early mathematics skills on later mathematics performance has been widely recognised. Review studies of mathematics interventions indicate that the interventions have mainly focused on school-aged children performing low in mathematics or with learning disabilities. These reviews have identified several beneficial instructional features (e.g., explicit instruction) that can be used in teaching mathematics to students with low performance in mathematics. Similar reviews on early mathematics interventions are lacking. In the light of the three-tiered model of educational support, there is a demand to provide the earliest possible support with evidence-based interventions in order to prevent the emergence and growth of difficulties. Concerning Finland, only a few research-based early mathematics interventions are currently available, thus, there is a need for developing and investigating new early mathematics interventions. This thesis aims to respond to these two gaps in the research field of mathematics interventions.

Consequently, the overall aim of this thesis is to investigate the effectiveness of early mathematics interventions for children with low performance in mathematics in the context of a three-tiered support model.

The specific aims were:

1. To review early mathematics interventions in terms of their effectiveness and pedagogical implementation.
2. To examine the effectiveness of kindergarten core mathematics instruction for children in general and in special support.
3. To develop and examine the effectiveness of second grade mathematics intervention in intensified support.

More specifically, Study I reviewed the early mathematics intervention studies conducted in the years 2000–2012. It focused on examining the effectiveness and pedagogical implementation (e.g., instructional features) of interventions. In Study II, the effectiveness of kindergarten core mathematics instruction (i.e., RightStart Mathematics) was examined in general education kindergarten groups, with a focus on low-performing children, and in Study III in the special education classes for kindergartners with a specific language impairment. In Study IV, a mathematics supplemental intervention for second graders performing low in mathematics was developed (i.e., Improving Mathematics Skills in the Second Grade) and its effectiveness was investigated. (See Table 2 for the detailed aims for each study.)

1.3.2 An overview of methodological solutions

A summary of the methodological solutions in the original studies is given in Table 2. As the methods are described in more detail in the original studies, only a brief overview of the methodological solutions is provided here.

The total number of participants in the review (Study I) was 2,711 children receiving intervention and 2,497 children serving as controls. In the intervention studies (Studies II and III), the number of kindergartners was 79, of which 47 received intervention. Nine of these children had a specific language impairment. A total of 88 second graders participated, of which 11 received intervention (Study IV).

The main measure assessing the early mathematics skills of kindergartners was the Early Numeracy Test (Van Luit, Van de Rijt, & Aunio, 2006) and in the follow-up, in first grade, BANUCA (BAsic NUmerical and Calculation Abilities, Räsänen, 2005). Futhermore, the children's addition fluency in general education kindergarten was measured using the Basic Addition Fluency Test (Salminen, Räsänen, Koponen, & Aunio, 2008). Raven's Coloured Progressive Matrices (Raven, 1965) was used to measure the children's nonverbal reasoning and the Peabody Picture Vocabulary Test–Revised (PPVT-R, Form L; Dunn & Dunn, 1981), using a shortened version adapted in Finnish (Lerkkanen et al., 2010), to measure receptive vocabulary. In the study focusing on second graders, mathematics skills were assessed using the Assessment of Mathematical Skills in the Second Grade (Aunio & Mononen, 2012). Second graders' thinking and language skills (reading fluency and comprehension) were assessed and used as measures of comparability among the groups before the intervention phase. For this purpose The Assessment of Thinking Skills in the Second Grade (ATS-2) (Hotulainen, Mononen, & Aunio, 2012) and ALLU: Ala-asteen lukutesti [ALLU – Reading Test for Primary School] (Lindeman, 2005) were used.

Study I applied different methodological solutions compared to studies II-IV because of its nature as a review study. In Study I, effect sizes for each mathematical performance dependent measure in the primary studies were calculated using standardised mean difference, Hedges' g, with correction for small sample sizes (see Turner & Bernard, 2006). In studies II-IV, pre-post control design was applied. The improvement within and between the (sub)groups was analysed (comparing the final status as well as the growth) using an analysis of variance (ANOVA) with Bonferroni-adjusted in multiple comparisons, and the non-parametric Mann-Whitney U-test, Wilson rank-sum, the Wilcoxon signed-rank and Kruskal-Wallis tests with small samples.

Table 2. A summary of the aims, participants, measures and analyses in the original studies

Study	Main aims	Participants	Measures	Main statistical methods	Purpose of the analysis
I	To examine the effectiveness and pedagogical implementation of early numeracy interventions (i.e., core and supplemental instruction) for four- to seven-year-old children at risk for mathematics difficulties.	19 studies: 5 core instruction (*n* = 1,904 children in experimental groups, *n* = 1,223 in control groups) and 14 supplemental instruction studies (*n* = 807 in experimental groups, *n* = 1,274 in control groups)	Data from the original studies	- Hedges' *g*	- Effect size
II	To investigate the effects of RightStart Mathematics (RS) instruction on kindergartners' learning of early mathematics skills compared to typical Finnish kindergarten mathematics core instruction (KLF).	Kindergartners RS group: 38 (6 LOW) KLF group: 32 (7 LOW)	- Raven - PPVT - The Early Numeracy Test - The Basic Addition Fluency Test - BANUCA	- Cronbach's alphas - ANOVA (Bonferroni-adjusted multiple comparisons) - Wilcoxon rank-sum and Kruskal-Wallis tests - Hedges' *g*	- Reliability of the scores - Differences in the background variables and effect of group - Confirming the results of small samples - Effect size
III	To examine the effect of RightStart Mathematics instruction on improving the early numeracy skills of kindergartners with a specific language impairment (SLI).	Kindergartners SLI: 9 NLP: 32 (i.e., KLF group in Study II)	- Raven - PPVT - The Early Numeracy Test - BANUCA	- Cronbach's alphas - Mann-Whitney U - Wilcoxon signed-rank test - Pearson's correlation coefficient (*r*)	- Reliability of the scores - Effect of group - Improvement within a group - Effect size
IV	To investigate how low-performing children receiving the Improving Mathematics Skills in the Second Grade intervention (LOWi) develop in their mathematics skills, compared to children in a control group (LOWc and TYPc)?	Second graders LOWi: 11 LOWc: 13 TYPc: 64	- The Assessment of Mathematical Skills in the Second Grade - The Assessment of Thinking Skills in the Second Grade - ALLU (reading fluency and comprehension)	- Cronbach's alphas - Kruskal-Wallis tests - Wilcoxon signed-rank test - Pearson's correlation coefficient (*r*)	- Reliability of the scores - Effects of group - Improvement within a group - Effect size

2 Overview of the original studies

This thesis consists of four studies. Study I reviewed the current state of early mathematics interventions and served as a starting point for conducting the three empirical intervention studies (Studies II-IV).

2.1 Study I

2.1.1 Aims

The purpose of Study I was to examine the effectiveness of early numeracy interventions for young children at-risk for mathematics difficulties (MD).[15] In addition, the study identified common pedagogical components (i.e., setting, duration, mathematics content used for intervention training and progress measurement, conductor and professional developmental support offered, and instructional design features) of interventions likely to influence their effectiveness.

2.1.2 Review methods

To identify a broad range of studies, searches in several educational databases were conducted, and references within articles were used in order to find more relevant studies. To be included in the review, the following criteria were set: (1) the study evaluated an early numeracy intervention programme for children who were at risk for MD in the age range 4–7 years, (2) the study used a random assignment or quasi-experimental design, (3) the study included a control group, (4) the sample size was allowed to vary, (5) a teacher or a member of the research team, such as a trained tutor, implemented the intervention, (6) the duration of the intervention was allowed to vary, (7) dependent measures included reliable quantitative measures of mathematical performance, (8) the study had to provide sufficient data for effect size calculations, and (9) the study could take place in any country, but it had to be published in English in a peer-reviewed journal between 2000 and 2012. Nineteen studies met these criteria. Accordingly, five studies were identified as core instruction interventions and fourteen as supplemental instruction interventions.

[15] In Study I, as in the following overview, term 'at-risk for mathematics difficulties' is used, to refer all children who perform below average mathematics skills (i.e., performing at or under the 25th percentile).

For each included study, the following information was coded: study characteristics (authors, year and country), methodological characteristics (design, measures, reliability and control group status), sampling characteristics (number of participants, mean age and at-risk status: LOW or SES),[16] and components of intervention (programme, duration, setting, leader, professional development provided, fidelity, instructional design features, materials, and practiced and measured numeracy content).

2.1.3 Analyses

Effect sizes for each mathematical performance outcome measure were calculated using Hedges' *g* with correction for small sample sizes (Turner & Bernard, 2006). The standardised mean differences in the outcomes of the experimental and control groups were calculated after adjustment for pre-test differences. Two studies used an analysis of covariance (ANCOVA), and, in these, Hedges' *g* was calculated as a covariate (pre-test) adjusted mean difference divided by the unadjusted pooled within-group post-test standard deviation (*SD*) (What Works Clearinghouse, 2008). In the fourteen studies that did not present adjusted means, but presented unadjusted pre-test and post-test means with *SD*s, the difference of the mean pre-post change in the experimental group and the mean pre-post change in the control group was calculated and divided by the pooled within-group pre-test *SD* (Morris, 2008). Three studies did not provide sufficient means or *SD*s, but had group equivalences on pre-test measures achieved through random assignment. With these studies, the presented t-test values were used to calculate Hedges' *g*. The confidence intervals (95%) for effect sizes were calculated by using the standard error of the effect size estimates (Turner & Bernard, 2006). The delayed effects of interventions, whether the intervention effect faded or continued were described, and statistical significances reported as they were given in the primary studies.

2.1.4 Results

Core instruction interventions

A total of 3,127 children participated in the primary studies (n = 1,904 in experimental groups, n = 1,223 in control groups). Sample sizes in the experimental groups ranged from 30 to 927 children. In the studies, two

[16] SES refers to children coming from families with a low socio-economic status.

programmes were used for 4–5-year-old children: Early Learning in Mathematics (ELM) and The Building Blocks (BB).

All core instruction interventions were implemented in whole group settings in classrooms. Common to all interventions was their long duration, from 25 to 26 weeks. The exact time used for practice varied from 2,470 minutes to 7,200 minutes. Both the BB and ELM programmes included a variety of early mathematical learning objectives for the targeted age group, such as recognition of numbers, object and verbal counting, comparison skills, ordering skills, simple addition and subtraction, place value knowledge, geometry and measurement. Teachers implemented all the core instruction interventions, replacing the mathematics instruction typically used in the classroom. The ELM programme applied explicit instruction with teaching concepts using CRA, mathematics-related discourse, and frequent and cumulative embedded reviews. Hands-on material and worksheets were primarily used. The BB programme shared some features (e.g., explicit instruction) with ELM. Unlike the ELM programme, the BB programme included more differentiation in classroom work by including small-group activities and individual computer work. Hands-on materials, books and games were also used in the BB. Children at risk for MD who received intervention made significant improvements in their learning compared to their controls. No follow-up of the development was included in any of the studies after the immediate post-test.

Supplemental instruction interventions

A total of 2,081 children participated in the primary studies (n = 807 in experimental groups, n = 1,274 in control groups). Sample sizes in the experimental groups ranged from 15 to 139 children. Three interventions used whole-group settings, in which instruction replaced some portion of typical classroom mathematics instruction. Seven interventions were delivered in small-group settings outside the classrooms. Two of these studies included individual work with computers. Four studies included one-to-one instruction, such as playing either a board game or a numeracy game on a computer. Intervention durations varied greatly: from 2 weeks to 36 weeks. Eight studies were short in their duration (*Mdn* = 3.5 weeks, range 2–8 weeks), and six studies were long (*Mdn* = 18.5 weeks, range 15–36 weeks). The median total time in minutes used in short interventions was 225 minutes (range 60–720 minutes), and in long interventions 1,330 minutes (range 500–1,900 minutes).

The supplemental interventions were divided into two categories based on whether the intervention concentrated on practising a variety of mathematical skills (9 studies) or a few specific skills (5 studies). Among multi-skill interventions, the six most practised skills were recognition of numbers, object counting, verbal counting, comparison, ordering, and early addition and subtraction. The activities generally operated within the 1–20 number range. In interventions focusing on specific mathematics skills, children practised addition and subtraction facts, and counting in computer-assisted instruction (CAI). The playing of a linear board game sought to promote children's numerical knowledge.

Teachers implemented five interventions and members of the research team implemented eight. In one study, both teachers and members of the research team were involved. Explicit instruction occurred in half of the interventions. One study combined guided instruction with explicit instruction. One intervention used peer-assisted tutoring, and one incorporated mathematical activities into daily routines. Four interventions (28.6%) used CRA. Games were used in five interventions (35.7%), and in three of these studies, a board game was used as the primary instructional material. Two interventions used only CAI and two other interventions included CAI as part of the instruction.

Although at-risk children made significant improvements in their learning, they were able to reach the performance level of their age peers in only three of the studies. Six studies provided follow-up information about children's development after the immediate post-test, and in five of them the intervention effect continued.

2.1.5 Discussion

Study I focused on providing evidence of the effectiveness of early numeracy interventions for young children at risk for MD, and identified the pedagogical components of the interventions likely to influence their effectiveness. The review yielded 19 intervention studies, which provided evidence that early numeracy interventions can improve the numeracy skills of young children at risk for MD. In the majority of the studies, the children receiving intervention outperformed the children in control groups, with the effects varying from small to large. Therefore, rather than waiting to provide effective mathematics interventions at school, evidence-based interventions could be used to promote early numeracy skills of children at risk for MD even before the onset of school. Progress in mathematics learning was evident when instruction included one or more of the following instructional features: explicit instruction, peer-assisted instruction, CRA, CAI or games; many of these were also found to be effective for low-performing school-aged children (e.g., Baker et al., 2002).

A variety of research designs guided the primary studies. Hence, interpreting and comparing the interventions' effects of the studies was not straightforward. None of the core intervention studies included delayed post measurements. Hence, there was no evidence to support whether the positive intervention effects lasted. In supplemental interventions, six studies applied a delayed post-test measurement after the intervention had finished. In five short intervention studies, the intervention effect continued when measured between three to nine weeks after the intervention. The longer the positive effect remains after the intervention, the more effective it is likely to be at preventing MD. Hence, it is important to include delayed post-tests in intervention studies.

As one of the aims of interventions is to reduce or even close the achievement gap, a group of typically performing (TYP) children, or standardised tests, should provide benchmarks for typical development, and should be included in intervention studies to show the performance level of LOW children compared to TYP children after the intervention. Although most of the core interventions boosted at-risk children's numeracy skills significantly and reduced the gap to age-related peers, only one core intervention study reported the percentage of at-risk children who reached age-level performance. Almost half of the supplemental intervention studies provided benchmarks if the at-risk children were able to reduce or close the performance gap between themselves and typically performing children. Despite the fact that the at-risk children were able to reduce the performance gap by making remarkable progress, several studies reported that they still lagged behind the performance level of their age related peers.

The positive intervention effects indicate that four- to five-year-old at-risk children's numeracy skills can be successfully promoted in core instruction. This has many benefits. If at-risk children's learning can be effectively promoted together by regular instruction instead of in pull-out lessons, only one teacher is needed to provide instruction. From the children's point of view, time for the intervention is not taken out of time allocated for other subjects, as can be the case in supplemental instruction. Although the instruction was given in a whole group setting, it did not mean that children were required to work as a whole group entity all the time. Children also had opportunities to work in small groups and in pairs, or individually with computers. On the other hand, working with a small group of children or one-to-one, a teacher has the opportunity to pay more attention to individual children's needs and to guide, model and give personal feedback.

In the primary studies, the members of the research team who conducted the interventions were additional resource personnel for preschools and kindergartens. One might question whether the interventions conducted by members of the research team would achieve similar magnitudes of effects if conducted by teachers, and whether it would even be possible to conduct the interventions with the school's current resources. Short supplemental interventions or using computers as intervention tools might be transferred to in-school intervention implementation with relative ease, whereas longer supplemental interventions often require additional personnel and resources.

To increase the validity of the review, strict criteria in selecting the studies was followed. The criteria were, however, not as tight as suggested by Slavin (2008). Including only research published in English might have overlooked some relevant studies. The review also excluded the results of intervention studies in dissertations and evaluations published in non-peer-reviewed journals.

In summary, the review found effective interventions to promote the numeracy skills of at-risk children already in early childhood education. As relatively few studies were found for this review, more intervention studies focusing on enhancing young children's numeracy skills are required in the future.

2.2 Study II

2.2.1 Aims

The purpose of Study II was to investigate the effects of RightStart Mathematics (RS) (Cotter, 2001) instruction on kindergartners' learning of early mathematics skills compared to typical Finnish kindergarten mathematics core instruction. In addition to comparisons between the two groups (i.e., all children), there was a focus on children who initially performed low in early mathematics skills. The mathematics performance of all the children was followed through to the first grade in order to see the long term effect of kindergarten instruction.

2.2.2 Participants and procedure

The participants were 70 kindergartners (43 boys and 27 girls) from three cities in southern Finland, ranging in age from 69 months (five years and nine months) to 81 months (six years and nine months). The RS group comprised 38 children and the group following the typical kindergarten mathematics programme (i.e., KLF group) 32 children. There were 13 children identified as performing low in mathematics (i.e., performing under the minus one standard deviation of the mean score of the Early Numeracy Test, ENT: Van Luit, et al., 2006), of whom six were in the RS group and seven in the KLF group. The six volunteered kindergarten teachers were all female, qualified kindergarten teachers and had several years of teaching experience (ranging from two to ten years). Before the instruction phase, all teachers were introduced to the assessment materials and the study procedure. In addition, the three RS teachers were briefly introduced to the RS programme and provided with a translated teacher's manual and the manipulatives required for implementing the programme.

A pre-test – instruction phase – post-test design was applied. The teachers in the RS group followed the RS programme for teaching mathematics for seven months during the academic year 2009-10, approximately two times a week, which replaced the typical mathematics instruction provided. The children in the KLF group received typical Finnish mathematics instruction (Kindergarten of The Little Forest, Wäre et al., 2009a, 2009b). Children's non-verbal reasoning skills and receptive vocabulary were measured before the instruction phase and mathematics skills before and immediately after the instruction phase, and in the first grade. The pre-test and immediate post-test were conducted either by a trained research assistant, teachers familiar with the tests, or the first author, in a quiet room in the kindergarten. The delayed post-test was conducted in the first-grade groups by the first author.

2.2.3 Measures

Raven's Coloured Progressive Matrices (Raven, 1965) was used to measure the children's nonverbal reasoning and the Peabody Picture Vocabulary Test–Revised (PPVT-R, Form L; Dunn & Dunn, 1981), using a shortened version adapted in Finnish (Lerkkanen et al., 2010), to measure receptive vocabulary. The Finnish Early Numeracy Test (ENT; Van Luit et al., 2006) was used for measuring children's early mathematics skills in kindergarten. There are two scales on the ENT: One measures relational skills, and one measures counting skills. The relational scale includes 20 items that measure comparison, classification, correspondence and seriation skills. The counting scale comprises 20 items that require the ability to use number words, synchronous and shortened counting, resultative counting and a general knowledge of numbers (see Aunio et al., 2006). The Basic Addition Fluency Test (Salminen et al., 2008) was used to measure kindergartners' early addition skills individually. There are 45 addition facts with numbers 1–5 presented horizontally in the test papers. The examiner shows a child one fact at a time and asks the child to give the answer to the problem. The test is time-limited (3 minutes). To measure mathematics performance in the first grade, BANUCA's (BAsic NUmerical and Calculation Abilities; Räsänen, 2005) five scales were used: number comparison, addition, subtraction, number words and arithmetic reasoning. A number comparison scale was also used in the kindergarten.

2.2.4 Analyses

The comparability of the RS and KLF groups was checked in relation to age and cognitive and mathematics performance at the pre-test time, using separate ANOVA tests. The instruction effectiveness from pre-test to post-test within and between the RS and KLF groups was analysed as performance growth, using separate ANOVAs. Then, the performance difference between the RS and KLF groups at a delayed post-test time was analysed. Similar analyses were conducted for the LOW groups (RS_{low} and KLF_{low}). Effect sizes were calculated for the mathematics outcome measures, using Hedges' *g* with a correction for small sample sizes. Non-parametric analyses (the Wilcoxon rank-sum test or the Kruskal-Wallis test) were conducted when the performance of small samples was compared, but if these analyses did not change significance in the findings, only the results from the parametric analysis were reported.

2.2.5 Results

Comparing the equivalency of the RS and KLF groups at the pre-test time, there were no significant differences between the groups in age, or on any of the cognitive and mathematics measures. The performance of both groups improved statistically significantly between the pre-test and the post-test on all the measured mathematics scales. Comparison of the gain scores from the pre-test to the post-test showed no statistically significant differences between the RS and KLF groups on any of the mathematics scales. Accordingly, the mathematics scores of both groups had improved similarly regardless of the type of kindergarten mathematics programme used. In the first grade, no performance differences on the BANUCA were found between the RS and the KLF groups.

The performance of both the low-performing groups (RS_{low} and KLF_{low}) improved statistically significantly between the pre-test and the post-test time on the ENT counting and addition scales. The counting skills in both groups improved to the same level as that of typically performing peers. Comparing the gain scores from the pre-test to the post-test, no statistically significant differences between the RS_{low} and KLF_{low} groups were found on any of the mathematics scales. In the first grade, no performance differences on the BANUCA were found between the RS_{low} and the

KLF_{low} groups, but the RS_{low} group performed more weakly than children in typically performing groups.

2.2.6 Discussion

The purpose of Study II was to investigate the effects of the RS programme on the learning of early mathematics skills of kindergartners, compared to typical Finnish kindergarten mathematics core instruction (the KLF programme). The RS instruction seemed to be as effective as the KLF instruction. The early mathematics skills (i.e., verbal and object counting, number comparison and addition facts knowledge) of all the children improved significantly in kindergarten regardless of the programme used. In the first grade, no difference in mathematics performance was found between the RS and KLF groups. Study II provided further evidence that mathematics core instruction can improve the mathematics skills of low-performing kindergartners (Chard et al., 2008; Clarke et al., 2011). In this study, the counting skills (both verbal and object counting skills) of the low-performing children improved to the level of their typically performing peers. As there were performance differences between the low- and typically performing groups in Grade 1, this result highlights the importance of being continuously aware of the children's mathematics performance level, and of providing them with opportunities for slowing down and practising skills that are challenging (L. S. Fuchs, Fuchs, Schumacher, & Seethaler, 2013).

The similarity of the results regardless of the programme used might be explained by the following factors. All the participating teachers had voluntarily attended the same in-service professional developmental course on early mathematics development before our study. Thus, they shared a positive attitude and a good knowledge of early mathematics building blocks and the teaching of mathematics, which have been shown to affect children's learning positively (Ertle et al., 2008; Sarama & Clements, 2009). The types of instructional features and content of the programmes might have been too similar in order to reveal performance differences between the groups. Furthermore, the instruction phase was long in duration.

The major limitation in Study II was the small number of participating children, especially in terms of low-performing children. Including only a small number of participants was partly due to limited resources: long core instruction studies with an adequate sample and a randomised design would be expensive to conduct. In future studies, observations in classrooms should be included, in addition to teacher-reported logbooks, to provide more information about how teachers implement the programme in practice.

2.3 Study III

2.3.1 Aims

The first aim of Study III was to examine the effect of RightStart Mathematics (RS) instruction on improving the early numeracy skills of kindergartners with a specific language impairment (SLI). The second aim was to investigate the extent to which children with SLI and their normal language-achieving age peers (NLP) differ in mathematics skills in kindergarten and Grade 1.

2.3.2 Participants and procedure

The participants in the RS group were nine children (seven boys and two girls) with a diagnosed SLI, from two kindergarten groups attending the same special state school for children with a SLI in central Finland. The children's ages ranged from 73 months (six years and one month) to 99 months (eight years three months). The reference group (NLP) consisted of 32 normal language-achieving kindergartners (21 boys and 11 girls) from two general education kindergarten groups from two cities in southern Finland, ranging in age from 69 months (five years and nine months) to 80 months (six years and eight months). The NLP group was the same as in Study II, in which it was referred to the KLF group. The RS teachers were two qualified female special education teachers with several years of teaching experience with children with a SLI. The three NLP teachers were female, qualified kindergarten teachers and had several years of teaching experience (ranging from six to nine years). Before the instruction phase, all teachers were introduced to the assessment materials and the study procedure. In addition, the RS teachers were briefly introduced to the RS programme and provided with a translated teacher's manual and the manipulatives required for implementing the programme.

The procedure for Study III was similar to that of Study II. The special education teachers for the SLI groups followed the RS programme and the children in the NLP group received business-as-usual mathematics instruction.

2.3.3 Measures

As in Study II, Raven's Coloured Progressive Matrices (Raven, 1965) were used to measure the children's nonverbal reasoning and the Peabody Picture Vocabulary Test–Revised (PPVT-R, Form L; Dunn & Dunn, 1981), using a shortened version adapted in Finnish (Lerkkanen et al., 2010), to measure receptive vocabulary. The Finnish Early Numeracy Test (ENT; Van Luit et al., 2006) was used to measure the children's early mathematics skills in kindergarten. In Grade 1, BANUCA's (BAsic NUmerical and Calculation Abilities; Räsänen, 2005) five scales were used: number comparison, addition, subtraction, number words and arithmetic reasoning.

2.3.4 Analyses

Owing to the small sample of children with a SLI, non-parametric tests to analyse the data were used. The performance growth of the children with a SLI on the ENT was analysed as a group as well as individually. At the group level, gain score comparisons between the SLI and NLP groups were used to measure the effect of the instruction. Immediate post-test and delayed post-test comparisons were used to show if the children with SLI had reached the performance level of the NLP children after the instruction phase and in the first grade.

2.3.5 Results

At the pre-test time, the NLP group significantly outperformed the SLI group on the ENT. The SLI group showed significantly more improvement than the NLP group on the ENT counting scale and there was no statistically significant difference between the groups on the ENT counting scale at the post-test time. At the individual level, most of the children with a SLI benefited from the RS instruction and showed age-level or near age-level performance on the ENT relational and counting scales right after the instruction phase. However, some children with a SLI did not respond to the instruction, and remained low performers compared to their age peers. In Grade 1, the children with a SLI performed similarly to their peers in addition and subtraction skills (accuracy) and multi-digit number comparison, but showed weaker skills in arithmetical reasoning and in matching spoken and printed multi-digit numbers.

2.3.6 Discussion

The purpose of Study III was to investigate the RS programme's effect on improving the early numeracy skills of kindergartners with a SLI at the individual and group levels. Also of interest was the extent to which children with a SLI differed in mathematics skills in kindergarten and first grade compared to their normal language-achieving peers who received business-as-usual mathematics instruction. Although the children with a SLI began kindergarten with significantly weaker early numeracy skills compared to their peers, they improved their counting skills to the level of their peers after the RS instruction ended. In the first grade, the children with a SLI performed similarly to their peers in addition and subtraction accuracy and multi-digit number comparison, but showed weaker skills in arithmetical reasoning and in matching spoken and printed multi-digit numbers.

Since transparent number-naming (e.g., "fourteen" is first practiced as "ten-four") with supporting visual material was emphasised in RS, the results of this study indicate that it is a beneficial method for teaching 'teen' numbers to children with a SLI. Moreover, non-counting strategies and the systematic visualisation of quantities with manipulatives, such as abacuses, may have supported the learning of counting skills. Through visualisation and working with tangible objects, children with a SLI may have been able to reduce their working memory load and were perhaps able to rely more on their stronger visuospatial sketchpad component. Since core instruction in a small group did not benefit all the children with a SLI, some children might need more intensified one-on-one tutoring in mathematics. The first grade mathematics performance indicated that the counting skills the children with a SLI had acquired during kindergarten were not strong enough for first-grade mathematics learning that required verbal processing.

The limitations of Study III were mainly related to the small sample size, the challenges of applying the study design to a population with learning difficulties, the use of various measurements and fidelity issues. The study did not include a control group of children with a SLI. In future studies, more cognitive (e.g., working memory) and language measurements should be included not only in the pre-test but also at later measurement points to see how possible gains in cognitive and language skills predict gains in mathematics performance.

2.4 Study IV[17]

2.4.1 Aims

The main aim of Study IV was to investigate whether low-performing second graders' mathematics skills could be improved using a novel, intensified mathematics intervention, Improving Mathematics Skills in the Second Grade (IMS-2) (Mononen & Aunio, 2012).

2.4.2 Participants and procedure

The participants were 88 second graders (48 boys and 40 girls) from four classes in schools in two southern Finnish cities, with mean age of eight years and two months (*SD* = 3.6 months) at the beginning of the school year. Twenty-four children were identified as performing low in mathematics (i.e., scoring at or below the 25th percentile in the mathematics pre-test), of whom 11 (four boys and seven girls) received IMS-2 instruction (LOWi) and 13 (four boys and nine girls) were in the control group (LOWc). The rest of the children served as a typically performing comparison group (TYPc). The interventionists were the first author (with special education teacher qualification and six years of teaching experience) and one female qualified special education teacher with five years of teaching experience. The teacher attended a six-hour training day before the intervention, which included an introduction to the study procedure and to the assessment materials and the intervention programme.

The first author and one trained teacher conducted all the assessment and intervention sessions. Mathematical skills were assessed three times: just before the instruction phase, immediately after and three months after the instruction phase. The thinking skills and language skills were assessed only at the pre-test time. The assessment sessions were held during the school day and required approximately three 45-minute sessions on separate days at the pre-test time, and a one 45-minute session at the post- and delayed post-test times. Intervention sessions were conducted in small groups of five to six children, in twelve 45-minute sessions over eight weeks.

2.4.3 Measures

Mathematics skills were assessed using the Assessment of Mathematical Skills in the Second Grade (Aunio & Mononen, 2012). This group-based

[17] The study was conducted in the ThinkMath project funded by the Finnish Ministry of Education and Culture (2011-2015).

paper-and-pencil test includes tasks of the early mathematics core skills in the number range of 1–1,000: number word sequence skills forward and backwards, base-10 and place value knowledge, addition and subtraction word problems, and multi-digit addition and subtraction calculations. Single-digit addition and subtraction facts in the 1–20 number range were also assessed (with a two-minute time limit).

Thinking skills were assessed using the Assessment of Thinking Skills in the Second Grade (Hotulainen et al., 2012). This group-based paper-and-pencil test includes tasks measuring two types of inductive reasoning skills: comparing attributes and comparing relations. Each item is a multiple-choice task, in which verbal instruction is given.

Reading fluency and comprehension were assessed using parts of the standardised second grade reading test, ALLU (Lindeman, 2005). In the reading fluency part, a child has to read four sentences and choose the correct one corresponding to its picture. A two-minute time limit is imposed to answer as many items (maximum of 20) as possible. In the reading comprehension part, a child reads a short non-fiction text and answers 12 multiple-choice questions (four possible answers in each). There is no time limit for the task.

Data on the children's home language and parents' educational level were collected by a questionnaire sent to the parents.

2.4.4 Analyses

First, the effect of gender of the child and the parents' educational level on the pre-test mathematics performance was examined. Furthermore, the performance in mathematics, language and thinking skills at pre-test time between the three groups (LOWi, LOWc and TYPc) were compared. Second, the intervention effects in terms of improving performance within each subgroup and the gain scores between the subgroups from pre-test to post-test and to delayed post-test time were compared.

2.4.5 Results

At pre-test time, boys and girls performed equally well on all pre-test measurements. There were no significant differences among the subgroups regarding the parents' educational levels. At pre-test time, the TYPc group outperformed the LOWi group in all measurements, except in reading fluency. No differences were found between the TYPc and LOWc groups on the scales measuring addition and subtraction facts, thinking and reading comprehension skills. Post hoc pairwise comparisons showed no differences between the LOWi and LOWc groups in any of the pre-test measurements.

All three groups made a significant improvement in their performance from pre-test to post-test in all the mathematics measurements (except LOWc in addition facts). The LOWi and LOWc groups did not differ in their gain scores from pre-test to post-test in any of the mathematics measurements. Compared to the TYPc group, the LOWc group improved more in its combined mathematics score. In the addition and subtraction facts measurements, the gains among the three groups did not indicate statistically significant differences. The LOWi group showed significant improvement from the pre-test to the delayed post-test only on the combined mathematics scale. The addition and subtraction facts scores of the LOWi children had decreased after the intervention had finished. The LOWc and TYPc groups showed significant improvements from the pre-test to the delayed post-test in all of the measurements.

2.4.6 Discussion

The purpose of Study IV was to investigate whether low-performing second graders' mathematics skills could be improved using the Improving Mathematics Skills in the Second Grade intensified instruction programme. In two months, those receiving intervention improved significantly in their mathematics skills, but their improvement did not differ from that of the controls, as measured in the gain scores. Three months after the intensified support finished, the mathematics performance of the LOWi group was almost at a similar level, compared to its post-test time performance, but decreased in addition and subtraction fluency.

The reason for these findings may be that some children in the LOWc group also received additional support in mathematics from their teachers after the post-test, although not initially intended. However, neither of the LOW groups was able to close the performance gap between them and their typically performing peers on the combined mathematics scale. Another explanation might be that the LOWi group had more severe difficulties in terms of its mathematics skills, compared to the LOWc group; 90.9% of the LOWi group scored at or under the 15th percentile on the combined mathematics scale at pre-test, whereas only 38.5% of the LOWc group did. The feedback from the teachers' logbooks revealed that the children had difficulties in keeping up with the pace of the intervention. The IMS-2 mathematics programme did not meet all the needs of the children in the LOWi group, and some of them probably would have needed more practice in basic skills (e.g., number sequence and counting skills) in the 1–100 range, and a longer intervention time should have been provided.

The IMS-2 did not boost the mathematics performance of low-performing second graders as much as expected, but the study gave valuable information about the functionality of the programme's intensity and content. It is necessary to develop the IMS-2 in terms of its intensity and content: working on the 1–100 range should be emphasised, since the children were unable to keep up with the pace of the new concepts and activities in the intervention programme, and the number of game-like activities could be increased in the programme, because the interventionists indicated the children's interest and engagement in these activities. Increasing the instruction time (duration in weeks, as well as minutes per session) might lead to more positive improvements. Owing to limited resources, one major limitation in the study was the small number of children involved in the intervention. In future studies, observations or video recordings in classrooms should be added to the use of logbooks to provide more reliable information on whether the teachers implement the programme as intended, as well as how the children behave and respond to the instruction.

3 General discussion

The main aim of this thesis was to investigate the effectiveness of early mathematics interventions for children with low performance in mathematics. Consequently, a review of early mathematics interventions was conducted and interventions were examined in terms of their effectiveness and pedagogical implementation. Furthermore, the effectiveness of two intervention programmes was examined. RightStart Mathematics (Cotter, 2001) was implemented both in general education kindergarten core instruction and in special education classes for children with a specific language impairment. Finally, a mathematics intervention programme for low-performing second graders was developed (Mononen & Aunio, 2012) and its effectiveness was examined.

3.1 Main findings of the studies

The review of early mathematics interventions provided evidence that, in general, early mathematics interventions are effective in improving the mathematics learning outcomes of children with low performance in mathematics. In the majority of the intervention studies, the children receiving intervention outperformed the children in control groups, with the magnitude of effect sizes varying from small to large. Even though the analysis of the pedagogical implementation in the interventions was descriptive in nature, progress in mathematics learning was evident when the intervention programme included one or more of the following instructional features: explicit instruction, peer-assisted instruction, CRA, CAI or games. Many of these instructional features have been found to be essential in mathematics instruction for school-aged children performing low in mathematics or with learning disabilities, too (e.g., Baker et al., 2002; Gersten, Chard et al., 2009; Kroesbergen & Van Luit, 2003). Therefore, rather than waiting for failure and providing mathematics interventions in later grades, early mathematics interventions can be used to promote the early mathematics skills of children with low performance in mathematics even before the onset of school and in the early grades.

In kindergarten, at the level of general support, the RightStart Mathematics (RS) instruction was as effective as the typical Finnish kindergarten mathematics instruction provided in the comparison group. The early mathematics skills (i.e., verbal and object counting, number comparison and addition facts knowledge) of all the kindergartners improved significantly during the kindergarten year. Furthermore, the counting skills of the initially low-performing children had improved to the level of their typically performing peers in the end of the kindergarten. Following the children's development in early mathematics skills to the first grade revealed that children in both groups (i.e., RS and comparison) performed similarly. In the comparison of the subgroups' performance, there were performance differences between the low- and typically performing children again in the first grade.

There were promising results in terms of the effectiveness of the RS when the intervention was provided for kindergartners with a specific language impairment (SLI). In the beginning of the kindergarten year the children with a SLI lagged behind their normal language-achieving age peers in early mathematics skills, a result that has been found in previous studies as well (e.g., Kleemans et al., 2011). Children with a SLI typically experience severe difficulties in counting skills due to the language demands in counting activities (e.g., Fazio, 1994, 1996). In our study, the children receiving RS instruction improved their counting skills to the level of their peers in the end of the kindergarten. At an individual level, most of the children with SLI showed age-level or near age-level performance in relational and counting skills after the instruction phase. However, some children with a SLI were resistant to the instruction and performed less well in early mathematics skills throughout their kindergarten year. In the follow-up, in the first grade, the children with a SLI performed similarly to their peers in addition and subtraction skills (accuracy) and multi-digit number comparison, but showed weaker skills in arithmetical reasoning and in matching spoken and printed multi-digit numbers.

To respond to the need for mathematics intervention programmes for early grades, a mathematics intervention programme for second graders performing low in mathematics was developed. Against expectations, this intervention did not boost the mathematics skills of the children receiving intervention more than that of the children in the control groups. However, the study provided valuable information about the functionality of the programme's intensity and content that need to be taken into consideration in the future. For example, children would need more practice in counting and conceptual place value skills in the number range 1–100, because they were unable to keep up with the pace of learning new concepts and operating in the number range 100–1,000, as reported by the interventionists. A need to increase the intervention's duration was revealed, as the performance of the children in the intervention group decreased after the intervention had finished. In the end of the study, it became apparent that the low-performing children in the control group had also received some supplemental instruction from their teachers after the post-test, although not initially intended. Informing the participating teachers more clearly about the research procedure itself and the expectations placed on them during the study should have been taken into consideration more carefully.

3.2 Theoretical implications

This thesis demonstrated that, in general, early mathematics interventions are effective in promoting the early mathematics skills of children with low performance in mathematics. This is an encouraging finding, as early mathematics performance has been shown to influence later mathematics performance (e.g., Duncan et al., 2007; Hornung et al., 2014; Jordan et al., 2007, 2010; Krajewski & Schneider, 2009). In accordance with the three-tiered model of educational support, rather than waiting for failure, children identified as having low levels of performance should receive early mathematics support. Hence, strengthened early mathematics skills should assist children in having a more positive developmental path in mathematics skills. The intervention studies of this thesis (Studies II-IV) applied a longitudinal approach (i.e., including follow-up measures). Such approach in intervention studies has been rather limited so far, thus further research is needed concerning the longitudinal effects of early mathematics interventions. Second, the results expanded the current knowledge about the instructional features that are effective in teaching mathematics for young children with low performance in mathematics. Those instructional features (e.g., explicit instruction, peer-assisted instruction, CRA and CAI) that had been found to be essential instructional features in mathematics instruction for school-aged children performing low in mathematics or with learning disability (e.g., Baker et al., 2002; Gersten, Chard, et al., 2009; Kroesbergen & Van Luit, 2003), are also essential in mathematics instruction for younger children.

This thesis is one of the first in Finland to explore the effects of kindergarten mathematics core instruction on children's early mathematics performance, using the RS programme. Also of interest was the development of early mathematics skills in children performing low in mathematics, when provided with the RS core instruction as general support. The effect of the RS instruction was similar to the typical Finnish kindergarten mathematics instruction. All kindergartners' early mathematics skills improved significantly during the kindergarten year. A long instruction phase, using an active control group, and several mathematics skills included in the RS programme (Fischer et al., 2013), might have affected the results so that the RS instruction did not show more positive effect compared to the typical Finnish mathematics instruction. The core instruction programmes used here were sufficiently effective to improve the counting skills of initially low-performing children to the level of their typically developing peers. These findings support the suggestion (Clarke et al., 2011) that systematic mathematics core instruction, which includes various effective instructional features, can serve as a first approach towards improving the mathematics performance of kindergartners, including those with low levels of performance.

Mathematics interventions for children with a SLI has been under-researched, although there is strong evidence that these children often face difficulties in mathematics skills both in their early childhood (e.g., Fazio, 1994, 1996; Kleemans et al., 2011) and later on at school (Koponen et al., 2006). This research revealed that in the beginning of the kindergarten year, the Finnish children with a SLI had weaker mathematics skills compared to their non-SLI peers. One theoretically interesting outcome was that the early mathematics skills of children with a SLI could be improved using the RS programme. Non-counting strategies, transparent number naming and the systematic visualisation of quantities with manipulatives (such as using abacuses) seemed to support the learning of counting skills of children with a SLI. However, at an individual level the mathematics performance and responsiveness to instruction was found to be heterogeneous, as has been reported in previous research with older children (Koponen et al., 2006). Therefore, a more individualised instructional approach might be necessary for some children with a SLI already in kindergarten.

This thesis shed light on the importance of longitudinal designs in intervention research. The longer the children are able to improve their learning after the intervention has ended, and can keep up with the development of their peers, the more effective the intervention is. Study I revealed that follow-up measures were rarely used in the intervention studies. The follow-up measures in Studies II-IV provided important information about the children's mathematics development after the intervention had ended. The results from studies with kindergartners showed that their mathematics performance was not stable from the end of kindergarten to the first grade. Some of the initially low-performing children, whose counting skills had improved to the level of their typically performing peers by the end of kindergarten, were performing lower than their peers in the first grade. In the IMS-2 intervention for second graders, the performance of the low-performing children who had received intervention decreased in addition and subtraction skills during three months after the end of the intervention. Thus, continuous assessment is needed in order to see the longitudinal effect of the intervention, as well as adjusting the level of support according to the child's needs, in practice.

Finally, this thesis demonstrated the complexity of interpreting and comparing the effects of different interventions and doing intervention research in kindergartens and in schools. From a methodological point of view, the intervention studies are very heterogeneous. There are several issues that affect the interpretation and comparison of the effectiveness of interventions (Fischer et al., 2013), for example, the effect size of the intervention is affected by the chosen reference group (i.e., active or passive groups regarding instruction, type of participants). In other words, the intervention may produce a large effect size when it is compared to one type of reference group but only a negligible effect when compared to another type of reference group. Furthermore, as emphasis is often placed on studies producing statistically significant results, a number of intervention studies may be hidden in the researchers' desk (i.e., the file-drawer effect; Ellis, 2010). Non-significant intervention results can, however, show which interventions, and more specifically what kind of elements in the intervention, are not effective enough to improve learning outcomes. These results may be used to guide the development of novel intervention programmes or to improve existing ones. Finally, experiences from doing intervention research in schools were gained from Studies II-IV (as well as from other on-going mathematics intervention research in the ThinkMath project, Mononen & Aunio, 2014). There are several important issues that have to be considered carefully in order to have a successful study when conducting intervention research. These include recruiting the participants (both teachers and students), co-operation with teachers, choosing an appropriate intervention programme and assessment tools (or in some cases researchers develop them by themselves), conducting the assessments and implementing the intervention in a certain time frame, and analysing the data and reporting the results. Regardless of careful planning, there are often variables in the research process that cannot be taken into consideration beforehand (e.g., some teachers may drop out from the study, misconceptions regarding the instructions) that may affect the initial plan and the results obtained.

3.3 Practical implications

The review of early mathematics interventions provides educators and researchers with an overview of and evidence about the effectiveness of interventions for children performing low in mathematics. Furthermore, results from the review suggest that including explicit instruction, peer-assisted instruction, CRA, CAI or games as part of early mathematics instruction for children with low performance in mathematics seems to be essential for improving the learning outcomes. Ideally, educators should have access to effective intervention programmes free of charge in order to provide equal learning opportunities for all children. Unfortunately, many intervention programmes are commercial and for one language, which may restrict the use and distribution of the programmes. In order to implement an intervention successfully, educators should be provided with professional developmental training concerning the theoretical background of the intervention and the principles for conducting the intervention (e.g., Clements, Sarama, Spitler, Lange, & Wolfe, 2011; Haseler, 2008). The requirement that teachers participate in face-to-face training should not become an obstacle for making an intervention programme as part of school practice. For some the cost of the training or the distance to the training place can be serious obstacles. To avoid this problem, for instance, in the ThinkMath project, the theoretical background and any information required to conduct the intervention will be provided for educators on the project's website as video clips and in handbooks free-of-charge, as are the intervention materials. The goal is to provide educators across Finland with an easy and stimulating way of accessing the intervention materials.

One practically relevant outcome is that the RS instruction seemed to be beneficial in boosting the counting skills of kindergartners performing low or with a SLI to the level of their typically performing peers. Even though Finnish RS material is not available at the moment for educators, it might be useful to integrate some features of the RS programme into mathematics instruction for kindergarteners performing low or with a SLI. Using non-counting strategies and a systematic visualisation of quantities with manipulatives, such as abacuses, seems to be beneficial for learning counting skills. Through visualisation and working with manipulatives, children may reduce their working memory load, an area in which these children often have limitations (Montgomery et al., 2010; Toll & Van Luit, 2014). The results also indicated that transparent number-naming (e.g., "fourteen" is first practised as "ten-four") with supporting visual material might be a beneficial method for teaching teen numbers. Applying CRA and guidelines in explicit instruction would be valuable, and are in line with the findings from the review study. Even though the RS was initially developed to be used as a core instruction programme in general education, it seems to be suitable for use in small-group instruction with kindergartners with a SLI.

3.4 Limitations and suggestions for future research

There were a number of limitations in the original studies, as well as, remarks made when conducting intervention studies that should be taken into consideration in future studies, in order to increase the quality of the research. These concern methodological, practical and ethical issues. Study I showed that there are a number of different ways to conduct and report an intervention study. Using different methodological solutions (e.g., concerning control groups, measurements, measurement times and analyses), practising and measuring a variety of mathematics skills, and using different cut-off criteria for the identification of children performing low in mathematics, meant that the interpretation and comparison of the intervention effects between the primary studies was not straightforward. In future, by conducting a meta-analysis, a range of different research designs might be taken more carefully into account and thus clarify the role of different instructional features that influence the effectiveness of early mathematics intervention. In Study I, rather strict criteria were used when selecting original studies for the review, in order to increase the validity of the review. In future, the scope might be broadened to include intervention studies in dissertations and evaluations published in non-peer-reviewed journals in order to reduce the likelihood of the file-drawer effect, as publication policies tend to favour studies reporting statistically significant results (Ellis, 2010). In general, researchers should ensure that they report all relevant information concerning an intervention so that it can be included in future reviews and meta-analyses. It is important to report information about the participants in different groups (e.g., background descriptives, identification criteria used low performance or mathematics learning disability), descriptions of the measurements used and the intervention programme, procedure (also in control groups), fidelity, and to provide adequate quantitative data for effect size calculations.

Studies II-IV showed the complexity of conducting intervention studies in kindergartens and in schools. The major limitations in Studies II-IV concerned the small number of participants, especially in terms of low-performing children, and a lack of randomisation for the intervention and the control groups. Ideally, in addition to the intervention group, the study design should include different types of control groups, such as active and passive controls (e.g., reducing the effect of additional attention given to the children in the intervention group). Therefore, the findings from Studies II-IV should be interpreted with caution. To address these shortcomings, further research investigating the findings in more detail, by employing larger samples and randomisation of groups, is warranted. On the one hand, the resources available limited the number of participants that could be included in the study. This affected especially Studies II and III, as these studies were conducted without specific funding (except for the purchase of manipulatives), and the first author, responsible for the research, was involved at the time of active conduction (i.e., intervention phase and data collection) in another full-time work. On the other hand, recruiting volunteer teachers with their students (especially a specific population, e.g., children with special needs), and randomly assigning the children to groups that will or will not receive intervention, is challenging. Furthermore, the age range of the children in the special education classrooms can be wide, as was the case in Study III, in which some older children were in their second year of kindergarten education. Therefore, these older children did not match the age range of kindergartners in general-education kindergarten classes.

Conducting intervention research in kindergartens and in schools adds ecological validity to the studies, but simultaneously makes it challenging to ensure that teachers conduct the intervention as initially intended. In Studies II-IV, only indirect measures (logbooks) were used (Gresham, MacMillan, Beebe-Frankenberger, & Bocian, 2000).The fidelity issue should be addressed more carefully in future intervention studies, in order to increase the validity of the intervention. Observations and video recordings in the classroom would provide more reliable information on whether the teachers implemented the programme as intended, and on how the children behaved and responded to the instruction. Considerations on the frequency of observations in the classrooms or of the video recordings, as well as deciding who would serve as the observer, would be needed in order to get a realistic picture of the implementation of the programme and of classroom behaviour. Furthermore, Study IV (as well as the ongoing mathematics intervention research in the ThinkMath project, Mononen & Aunio, 2014) indicated that there was a need to inform the participating teachers more clearly both about the research procedure itself, and regarding what was expected of them during the study. Even though a majority of the teachers were comfortable with adapting aspects of research into their teaching when participating in an intervention study, some teachers seem to need more guidance in following the instructions given by the researchers.

An ethical dilemma may arise concerning which low-performing children will receive the intervention in intervention studies. Teachers may feel worried if they have to leave some children without supplemental support during the intervention study. This became evident in the Study IV, as some children in the control group were provided supplemental instruction by their teachers, although not initially intended. One solution is to consider study designs that take this ethical issue into account.

Alternative study designs for those that were present in Studies II-IV would be to provide the same intervention for control children after the study has ended, use a two-period cross-over design (i.e., provide intervention for both groups so that one serves first as the intervention group and the other one as the control group and then vice versa), or to use a regression discontinuity design (e.g., Bryant, Bryant, Gersten, Scammacca, & Chavez, 2008) that allows all low-performing children to receive intervention. These alternative methods, however, may have problems. For example, the length of the intervention study may inhibit the provision of the intervention for the control group's children early enough (e.g., the intervention study with delayed post-test may easily require over half of the school year).

The non-significant intervention effects in Studies II and IV may partly be explained by the fact that both programmes included several mathematics topics to be practised. Interventions that would intensively target only one or two core early mathematics skills at a time might show better effects (e.g., Fisher et al., 2013). In the future, it would be of interest to investigate whether practising one particular skill (e.g., nonverbal number sense that has been suggested to lay a foundation for other mathematics skills) had a transfer effect to other skills as well. For example, a study by Wilson, Revkin, D. Cohen, Cohen, and Dehaene (2006) showed promising results that training nonverbal number sense skills with the adaptive computer game "The Number Race", improved not only the nonverbal number skills of children with low performance or with mathematics learning disability, but also subtraction accuracy. However, no improvement was found in addition and base-10 comprehension skills.

Serving as a whole year kindergarten programme, the RS programme included a variety of early mathematics skills. As the focus of this thesis was the development of early mathematics core skills, and due to the measurements selected, some relevant information may have been missed concerning the development in other areas of mathematics, such as in geometry and measurement. It would have been interesting to see how the children performed in the nonverbal number sense (e.g., in subitising or ANS tasks), but such assessment tasks were not available in Finland at that time. The IMS-2 intervention focused on early mathematics core skills, namely practising counting and conceptual place value. As the results showed, the IMS-2 intervention programme needs modification concerning its content (e.g., working only in the number range 1-100) and duration (i.e., increasing the instruction time) and another experimental study to show if these modifications are enough to increase further the mathematics skills of low-performing second graders. It may also be possible that the measure used was not sensitive enough to reveal the changes in the development, because of the number range of the test (1-1,000) and the combined score of the test was used.

Domain-general cognitive skills, such as working memory and language skills, have been shown to be related to mathematics development (e.g., Friso-van den Bos et al., 2013; LeFevre et al., 2010) but were only partly investigated here. The cognitive skills measured in this thesis, such as thinking skills and language skills, were used only as measures of comparability among the groups before the intervention phase. In future studies, more cognitive skills measurements should be included not only in the pre-test time but also at later measurement points to see how performance and possible gains in cognitive and language skills predict gains in mathematics performance. From the young child's perspective, the number of different tests and the time the tests take to accomplish should be taken into consideration when planning the assessment test battery, so that the child would not feel exhausted as a result of the number of tests and that the test results would show the child's realistic performance.

One of the major goals of early mathematics support is to diminish or prevent the emergence of mathematics learning disability later. Only following several years those children who have received early mathematics intervention would provide sufficient evidence of the impact of early interventions in the long run. The effect of one intervention programme might be difficult to demonstrate in longitudinal studies, if the child had received different types of intervention over the years. However, this kind of design might inform us about the variety and intensity of support that children need in their mathematics development. The review of interventions revealed that follow-ups were missing in several intervention studies. In Studies II and III, the children were followed for half a year and in Study IV for three months after the intervention finished. Relatively short follow-up periods showed that for some children the intervention was not enough to keep up with the development of their peers after the intervention had ended, and that they would need continuous support and repetition in early mathematics core skills.

There is evidently a need for evidence-based mathematics interventions in kindergarten and school practice, but providing such interventions is a challenge for researchers. There are a number of different combinations to investigate simply with regard to different mathematics skills, different subgroups of children (e.g., age, specific disability groups etc.), and study designs. Thus, it is reasonable to ask whether it is even realistic to expect that a majority of mathematics interventions in school practice will someday be evidence-based. The stricter the criteria set for the definition of evidence-based practice, the slower the inclusion of evidence-based interventions in everyday school practice. The intervention studies of this thesis do not meet all the criteria set for evidence-based interventions (Brown-Chidsey & Steege, 2005) For instance, the samples of low-performing children were small, the fidelity issue was only narrowly addressed, and there was a lack of SLI controls in Study III. Furthermore, only Study III showed a significant intervention effect. The 'evidence-informed' approach (Hammersley, 2013) might serve educational interventions better than an 'evidence-based' approach, giving educators the opportunity and responsibility to interpret and evaluate the evidence of the intervention (also the non-significant results) in relation to their current practice.

3.5 Conclusions

To conclude, the results of this thesis indicate that, in general, mathematics interventions can be used successfully to promote the early mathematics skills of children with low performance in mathematics even before the onset of school and in the early grades. Rather than waiting for children to fail, the identification of children performing low in mathematics should be emphasised in early childhood education and sufficient support provided for those children, according to the three-tiered model of educational support. The early mathematics support given may diminish and prevent the emergence of mathematics learning disability. However, more research evidence is needed of the longitudinal effects of the early mathematics interventions. Educators should have easy access to mathematics intervention programmes that have been shown to improve mathematics learning outcomes. Currently, the number of such programmes is limited in Finland and therefore further research in the area is needed.

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Appendix

A summary of review and meta-analysis studies examining mathematics interventions.

Reference	Title	Publication years of primary studies in the review	Number of intervention studies	Number of participants	School level range	Type of participants	Type of mathematics skills practised in interventions	Key findings with reported effect sizes (*d*, if not otherwise stated [a])
Baker, Gersten & Lee (2002)	A synthesis of empirical research on teaching mathematics to low-achieving students	1971–1999	15 group-design	not reported	grades 2–11	low-achieving or at risk for failure	a range of mathematics skills	Effective features: (1) providing data on student performance, 0.57, (2) peer-assisted learning, 0.66, (3) providing clear, specific feedback to parents on their children's mathematics success, 0.42, (4) using principles of explicit instruction, 0.58

Reference	**Title**	**Publication years of primary studies in the review**	**Number of intervention studies**	**Number of participants**	**School level range**	**Type of participants**	**Type of mathematics skills practised in interventions**	**Key findings with reported effect sizes (*d*, if not otherwise stated [a])**
Codding, Burns & Lukito (2011)	Meta-analysis of mathematic basic-fact fluency interventions: a component analysis	1989–2007	17 single-subject	55	elementary	performing below expectations in mathematics (i.e., having mathematics disability or other special educational needs)	computation (addition, subtraction, multiplication, division)	Effective features (phi coefficient): (1) practice with modelling, 0.71, (2) drill, 0.92, (3) self-management (i.e., self-instruction), 0.55
Codding, Hilt-Panahon, Panahon & Benson (2009)	Addressing mathematics computation problems: A review of simple and moderate intensity interventions	1980–2007	12 group-design, 15 single-subject	914	kindergarten to grade 12	needing additional support in mathematics	computation (addition, subtraction, multiplication, division)	Effective features: (1) simple interventions: earning free time, flash cards, cue-cards, count-bys, large effect sizes, (2) moderate interventions: self-instruction, taped problems, incremental rehearsal, large effect sizes

Reference	Title	Publication years of primary studies in the review	Number of intervention studies	Number of participants	School level range	Type of participants	Type of mathematics skills practised in interventions	Key findings with reported effect sizes (*d*, if not otherwise stated [a)])
Fischer, Moeller, Cress & Nuerk (2013)	Interventions supporting children's mathematics school success	2000–2012	39 group-design	8,991	preschool to secondary	typically, low-performing and having mathematics disability	a range of mathematics skills (e.g., arithmetic operations, magnitude comparison, word problems, base-10 system)	(1) Studies that did not employ a performance-matched control group produced larger effect sizes than those that did, (2) studies using an alternative intervention that was comparable to the experimental intervention led to a decrease in effect size, (3) studies consisting of one component (i.e., training one specific mathematics skill) produced significantly greater effect sizes than multi-component (i.e., covering more than one mathematics skill) studies, (4) sample and methodological characteristics (e.g., grade level or intervention duration) were not associated with effect sizes

Reference	Title	Publication years of primary studies in the review	Number of intervention studies	Number of participants	School level range	Type of participants	Type of mathematics skills practised in interventions	Key findings with reported effect sizes (*d*, if not otherwise stated [a)])
Gersten, Chard, Jayanthi, Baker, Morphy & Flojo (2009)	Mathematics instruction for students with learning disabilities: A meta-analysis of instructional components	1971–2007	42 group-design	not reported	school-age students	learning disabilities	a range of mathematics skills categorised as operations, word problems, fractions, algebra, general math proficiency	Effective features: (1) use of heuristics to solve problems, 1.56, (2) explicit instruction, 1.22, (3) visual presentation, 0.47, (4)student verbalisations, 1.04, (5) providing teachers feedback about student performance, 0.23
Kroesbergen & Van Luit (2003)	Mathematics interventions for children with special educational needs	1985–2000	58 group- and single-subject designs	2,509	kindergarten to elementary school	special educational needs in mathematics (incl. low-performing and at-risk in mathematics)	preparatory mathematics (i.e., early mathematics skills), basic skills (e.g., addition, subtraction, multiplication and division facts), problem-solving strategies	The interventions concerning basic skills showed the highest effect sizes, 1.14. Effective features: (1) explicit instruction, 0.91, (2) self-instruction, 1.45, (3) computer-assisted instruction, 0.51

Reference	Title	Publication years of primary studies in the review	Number of intervention studies	Number of participants	School level range	Type of participants	Type of mathematics skills practised in interventions	Key findings with reported effect sizes (*d*, if not otherwise stated [a)])
Kunsch, Jitendra & Sood (2007)	The effects of peer-mediated instruction in mathematics for students with learning problems	1980–2004	17 group-design	460 with disabilities, 643 at risk, 627 without disability	elementary to secondary	learning disabilities or at risk for mathematics disabilities (e.g., low-performing)	peer-mediated instruction in mathematics (computation: facts and procedures or computation, concepts and application)	Peer-mediated interventions in mathematics were moderately effective for elementary-aged students, 0.57, and students at risk for mathematics disabilities, 0.66, and less effective for secondary students, 0.18, and students with disabilities, 0.21. Overall effect size 0.47.
Maccini, Mulcahy & Wilson (2007)	A follow-up of mathematics interventions for secondary students with learning disabilities	1995–2006	13 group-design, 10 single-subject	1,134 (384 with learning disability)	grades 6–12	learning disabilities	computation (addition, subtraction, multiplication, division), fractions, decimals, geometry, word problems, integers, algebra	Effective instructional features: (1) mnemonic strategy instruction, 1.68 - 2.22, (2) concrete-representational-abstract sequence, 0.43 - 3.87, (3) cognitive strategy instruction involving planning, -0.2 - 1.4, (4) schema-based instruction, 1.69,(5) contextualised videodisc instruction, 0.56 - 0.81. Explicit instruction was incorporated in effective interventions.

Reference	Title	Publication years of primary studies in the review	Number of intervention studies	Number of participants	School level range	Type of participants	Type of mathematics skills practised in interventions	Key findings with reported effect sizes (*d*, if not otherwise stated [a)])
Malofeeva (2005)	Meta-analysis of mathematics instruction with young children	1977–2003	29 group-design	1,845	preschool to kindergarten	typically and low-performing	early mathematics skills	Early mathematics instruction was effective, 0.47. Effective features: (1) combination of guided and explicit instruction, 0.63
Methe, Kilgus, Neiman & Riley-Tillman (2012)	Meta-analysis of interventions for basic mathematics computation in single-case research	1999–2011	11 single-subject	47	kindergarten to elementary school	learning disabilities or low-performing	computation (addition and subtraction)	Overall effect sizes (IRD) ranged from moderate, 0.59, to high, 0.86. Effective instructional features: (1) practice under speeded conditions, 0.97, (2) concrete-representational-abstract sequence, 0.94

Reference	Title	Publication years of primary studies in the review	Number of intervention studies	Number of participants	School level range	Type of participants	Type of mathematics skills practised in interventions	Key findings with reported effect sizes (*d*, if not otherwise stated [a)])
Miller, Butler & Lee (1998)	Validated practices for teaching mathematics to students with learning disabilities: A review of literature	1988–1997	54 group-and single-subject designs	1,034	elementary to secondary	learning disabilities	a range of skills (e.g., addition, subtraction and multiplication facts, place value, number concept, money, long division, number names)	Effective features: (1) use of strategies (step-by-step processes) (2) use of self-monitoring (e.g., checklists), (3) use of manipulatives and drawings, (4) explicit instruction, (5) peer tutoring, (6) computer-assisted instruction. Effect sizes not reported.
Xin & Jitendra (1999)	The effects of instruction in solving mathematical word problems for students with learning problems	1980–1996	14 group-design, 12 single-subject	653	elementary to post-secondary	disabilities and at risk for mathematics disabilities	word-problem-solving	ES for group designs interventions was 0.89 and 89% (PND) for single-subject designs. Effective features: (1) computer-assisted instruction, 1.80 (group designs), (2) representation techniques, 1.77 and 100% (group and single-subject designs), (3) strategy training (incorporating explicit instruction and/or metacognitive strategies), 0.74 and 87% (group and single-subject designs)

Reference	Title	Publication years of primary studies in the review	Number of intervention studies	Number of participants	School level range	Type of participants	Type of mathematics skills practised in interventions	Key findings with reported effect sizes (d, if not otherwise stated [a])
Zhang & Xin (2012)	A follow-up meta-analysis for problem-solving interventions for students with mathematics difficulties	1996–2009	29 group-design, 10 single-subject	not reported	kindergarten to grade 12	typically performing and with learning problems	word-problem-solving	Effect size for group-design interventions was 1.85 and 95% (PND) for single-subject designs. Effective features: (1) problem structure representation (explicit instruction and conceptual modelling are essential parts), 2.64, (2) cognitive strategy training, 1.86, (3) assistive technology, 1.22

Note. [a] Effect size interpretations. Cohen's *d*: 0.2 to 0.49 is considered as small, 0.5 to 0.79 is considered as medium and 0.8 and greater as large. A phi coefficient of 0.29 is considered as neglible, 0.30 to 0.49 as small, 0.50 to 0.69 as moderate and 0.70 or higher as strong. IRD of 0.49 is considered as a small effect, 0.50 to 0.69 as moderate and effect sizes larger than 0.70 as a strong effect. PND = percentage of non-overlapping data.